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CLASSICAL ELECTRODYNAMICS, A MIRACULOUS MIDWAY BETWEEN ELECTROTECHNICS AND QUANTUM THEORY

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Nearly a half of a century ago, when I have been designated to teach the theory of classical electrodynamics at the University Babes-Bolyai in Cluj, I had a strong feeling of discomfort. The reason was simple. On one side, I knew that classical electrodynamics, invariant to the Lorentz transforms, was the basis for the special relativity postulated by A. Einstein. On the other side, then, all the treatises, that I might use as models, followed the same pattern: from electrostatics and magnetostatics to Maxwell equations and, finally, the special relativity. I was afraid that my students would hardly eliminate from their brain the intuitive concepts used in the preliminaries, together with the mathematical analysis in spaces with definite metrics used extensively by Maxwell in building his system. My question was: how convincing could I be when after a pledge on the correctness of the Coulomb law to a level of 1/1 000 000 000 for the exponent of 2 in the denominator of the dependence on the distance, demonstrated in the experiments of Plimpton and Laughton or Lamb and Retherford, I am simple saying that at a dynamical approach Coulomb's law is false! Even more, how might I pretend the correctness, even at the dynamical level, of the divergence law as I knew that it is based on two incorrect dynamical concepts ? What does it means an electrical flux through a extended surface or the electromotric tension in an extended circuit? At what time should we take the intensity of the electric field in different points of the surface or contour? Obviously I can extend substantially the list of questions.

At the same time I must recognize that in practice, in electrotechnics, these concepts play an essential role and that our life is based on the successes of electrotechnics! At a laboratory scale, electrotechnics gives quite accurate answers, especially for fields of low frequencies or in absence of sudden temporary changes. Even in the most advanced laboratories many physicists are thinking, even today, in the spirit of the electrotechnical concepts and rules.

A particular role in the whole story has been played by the metallic conductors. On one side, due to the shortness of their relaxation times, their electrons are nearly instantaneously reaching stable equilibrium, metallic surfaces becoming equipotential. This encouraged the study of electromagnetic fields, in models with given equipotential surfaces. On the other side, due to the quasifree electrons on the conduction band, metallic conductors blurred the physical understanding of the local electromagnetic induction.

It is well known the extraordinary role played by Faraday in the development of the electrodynamics by his brilliant experiments on electromagnetic induction. Paying whole admiration for his merits, we should not forget that Faraday was primarily interested in solving an electrotechnical problem: how to obtain electrical currents with magnets. He obtained those currents by changing something in time, by moving conductors, by moving magnets and, essentially, by modifying along time the magnetic field crossing an immobile closed metallic circuit. In interpreting his results, quite naturally he used two concepts: magnetic flux and electromotric tension Why? Due to the fact that as he was an eminent scientist he knew that aside the spectacular success of obtaining the desired currents, he should establish a quantitative connexion between causes and effects. He was not contented with his important qualitative success, he was looking for a physical law. As far as I know, no other concepts, excepting the speed of the variation of the magnetic flux and electromotric tension, are more suitable for his scope. As he was experimenting with finite closed metallic circuits and with magnetic fields commonly produced by magnetic rods, He could measure precisely the variation of the magnetic flux and the intensity of the currents in the metallic circuit and establish the proportionality between them. We should not forget that everything has been obtained at a common laboratory scale.

The genial step forward made by Faraday was the extension his results obtained for mobile magnets or mobile conductors to the case, discovered by him, of inducing electromotric tension in an immobile metallic circuit by a time varying magnetic field. He introduced into the game the time! But we must be very cautious. His statement was a global one, made in the spirit of the current concepts of the Newtonian approach.

It is not the object of this essay to enumerate the merits of J. C. Maxwell in the elaboration of the present classical electrodynamics; there are many. One of the most prominent was the transcription in local terms of the global laws of the electrodynamics known at his time. For achieving that, he used extensively the very important formulas of the mathematical analysis in Euclidean spaces like flux-divergence law and Stokes theorem. It is important to insist on the fact that all theorems used by Maxwell have been established in Euclidean spaces. Excepting Ampère's circuital law all the formulas written by Maxwell were transcription in local terms of the well known global laws of electromagnetism, based on uncontroversial experiments. I do not insist on the mathematical formalism invented by Maxwell itself, important, but irrelevant for this discussion.

The reason for introducing by Maxwell of a supplementary source of the magnetic field in Ampère's circuital law related to the conservation of the electric charge is well known. It is important to insist that the profound motivation of this lengthy and severely contested step made by him is essentially related to the birth of the present day electrodynamics, the birth of the concept of electromagnetic field. It is an act of justice to claim that this step is comparable in importance with Faraday's discovery of the electromagnetic induction!

It is important to emphasize that Maxwell equations building together a necessary and sufficient system of defining a vectorial field in an Euclidean space, have been obtained in the purest spirit of the Newtonian mechanics! None of the steps persuaded in building this system have been made outside the mentioned spirit!

Surprisingly, this system of equations is defining an unphysical world as it is depending of the particular choice of the inertial frame , as it is not invariant to the Galilean transforms?! All the difficulties are steaming from the local version of Faraday's law and from the supplementary source of the magnetic field introduced by Maxwell, expressed by first time derivatives. This was a supplementary argument against this term added by Maxwell, especially as it did not have an experimental motivation. But all the consequences resulting from the equations were in full agreement with the experiments! Lorentz transforms, that mathematically leaves invariant the system in all the inertial frames, differs of Galilean ones!

As we know, the answer to this dilemma has been given by A. Einstein in his postulate on special relativity which point that there exists a maximal speed of transmitting the interaction, the speed of light in vacuum; Asides offering a solution to the above mentioned dilemma, Einstein theory gave a consistency to the concept of a physical field. But attention! Einstein's postulate says that the Newtonian approach is only approximately correct and at a rigorous level we may not neglect the retardations due to the finite speed of light! This means that whole concepts used before in electromagnetism are only approximately correct, or let us say, electrotechnically correct! At the same time, the whole procedure of deducing the local equations of the Maxwell system, based on traditional mathematics, is also only approximately correct. We are faced with a miracle: correct equations obtained from approximately correct concepts using an approximately correct procedure. Being more radical, we must recognize: the correct equations of Maxwell's system are based on incorrect concepts, and are obtained by incorrect procedures. The electromagnetic field exists and it is given by Maxwell system! But what are we doing with the fields E and B?

Significantly enough, the next step is a typical new Newtonian one; at least in spirit. We are running to write down the force which is acting on an electrical charge in an electromagnetic field: the Lorentz force! Now everything is all right. We have what to put in the right side in Newton's fundamental law of mechanics. The Newtonian mathematical machinery is ready to start. Even more, we invented marvellous instruments to measure electric and magnetic fields.

After the acceptance of the special relativity and the reconciliation of the Newtonian mechanics with relativity, it seemed that everything was returned to normality. The success of the electromagnetic theory of light completed the triumph of the classical electrodynamics. The game with Maxwell's equations which leaded to the quadrivector of electromagnetic potential remained a simple game despite the alarming signal pulled by Lorentz gauge.

The Hamiltonian (Lagrangian) formulation of the classical mechanics steamed from a very general variational principle was considered, a long period of time, at the beginning previous century, a mathematical preciosity. This feeling was strengthened by the fact that a simple correspondence with the Newtonian mechanics could be demonstrated (at least in many common cases). But the experiments, as well as a more careful consideration of the facts, were pushing, more and more, mechanics toward an unexpected direction. It was recognized that the Lagrangian contains more physical information than the force, used in the Newtonian fundamental law of mechanics.

But surprise: the Lagrangian (Hamiltonian) of the electrically charged particles in an electromagnetic field does not use electric or magnetic fields! It contains the electromagnetic potentials! No problem; by using liberties offered by gauges, we may build immediately the intensities of the electromagnetic fields at the end of the game. A long period of time the electromagnetic potentials were considered as useful mathematical instruments, nothing else.

The capital blow came from the discovery of quantum mechanics and then of quantum electrodynamics. We must emphasize that none of both used the concept of force, on contrary both used the concepts of Hamiltonian (Lagrangian). So, in the fundaments of the quantum theory are used potentials, not fields! It seems that sophisticated experiments (Bohm-Aharonov) have shown that electromagnetic potentials are not simple mathematical instruments; they have a deep physical meaning, are real physical parameters. Why are we leaded to consider that potentials are simple instruments, secondary ones? Because at a common laboratory scale we are measuring easily the intensities of the electric and magnetic fields. Those are our habitual parameters that we can accurately measure with our instruments. A conceptual similar situation is that of light. Did anybody say a simple word about the electromagnetic nature of the light before Maxwell has written his system? Obviously not, because we were not get used with the measurements of the rapidly varying electromagnetic fields. Nobody doubts about the merits of galvanometers, but nobody uses these sensitive instruments to measure the intensity of alternative currents, due to their inertia.

As a result of the previous considerations, we arrive at the conclusion that the Maxwellian classical electrodynamics, a miracle as I said, is a valuable theory at the macroscopic scale which deserved to justify the special relativity and gave a physical content to the concept of wave, one of the dual characteristics of our world. It is fair to say that it deserve to solve intricate problems of the common electrotechnics.