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## QUARTETTING IN FERMION MATTER AND ALPHA PARTICLE CONDENSATION IN NUCLEAR SYSTEMS

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*Abstract.* The famous Hoyle state ( $0_2^+$  at 7.654 MeV in  $^{12}\text{C}$ ) is identified as being an almost ideal condensate of three  $\alpha$ -particles, held together only by the Coulomb barrier. It, therefore, has a  $^8\text{Be}$ - $\alpha$  structure of low density. Transition probability and inelastic form factor together with position and other physical quantities are correctly reproduced without any adjustable parameter from our two parameter wave function of  $\alpha$ -particle condensate type. The possibility of the existence of  $\alpha$ -particle condensed states in heavier  $n\alpha$  nuclei is also discussed.

*Key words:* Bose condensation,  $\alpha$ -clusters, nuclear structure.

### 1. INTRODUCTION

Quantum condensation of particles is one of the most amazing phenomena of many body systems. Striking well known examples are superconducting metals and superfluid  $^4\text{He}$ . Also nuclei are superfluid. However, in nuclei the most tightly bound cluster is not a pair but a quartet. Therefore, what about  $\alpha$ -particle condensation in nuclei? The only nucleus which in its ground

state has a pronounced  $\alpha$ -cluster structure is  ${}^8\text{Be}$ . In Fig. 1(a) we show the

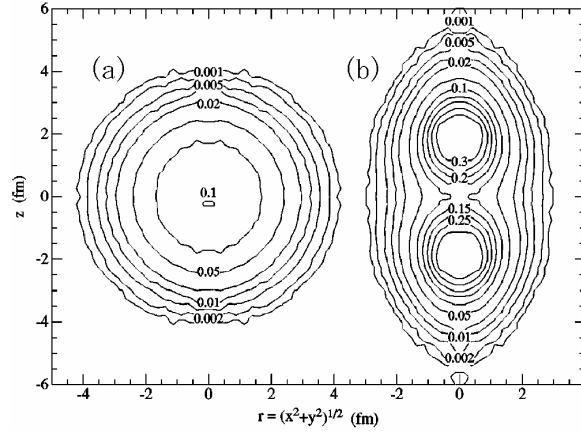


Fig. 1 – Contours of constant density (taken from [1]), plotted in cylindrical coordinates, for  ${}^8\text{Be}(0^+)$ . The left side (a) is in the “laboratory” frame while the right side (b) is in the intrinsic frame.

result of an exact calculation with a realistic  $N$ - $N$  interaction for the density distribution in the laboratory frame, whereas in Fig. 1(b) we see the same in the intrinsic, deformed frame where in addition the question has been asked where to find the second  $\alpha$ -particle when the first is placed at a given position. So we see that the two  $\alpha$ 's are  $\sim 4$  fm apart giving raise to a very low average density  $\rho \sim \rho_0/3$  as seen on Fig. 1(a), where  $\rho_0$  is the nuclear saturation density.  ${}^8\text{Be}$  is also a very large object with an rms radius of  $\sim 3.7$  fm to be compared with the nuclear systematics of  $R = r_0 A^{1/3} \sim 2.44$  fm. Definitely  ${}^8\text{Be}$  is a rather unusual nucleus. One may ask the question what happens when one brings a third  $\alpha$ -particle alongside of  ${}^8\text{Be}$ . We know the answer: the 3- $\alpha$  system collapses to the ground state of  ${}^{12}\text{C}$  which is much denser than  ${}^8\text{Be}$  and cannot accommodate with its small rms radius of 2.4 fm three  $\alpha$ -particles barely touching one another. One nevertheless may ask the question whether the dilute three  $\alpha$  configuration  ${}^8\text{Be}-\alpha$  may not form an isomeric or excited state of  ${}^{12}\text{C}$ . This will be the main subject of our considerations.

## 2. $\alpha$ -CONDENSATE STATES IN SELF-CONJUGATE $4n$ -NUCLEI

We now will show that  $\alpha$ -particle condensation most likely exists in  $n\alpha$  nuclei around energies of  $\alpha$ -particle break up thresholds. We want to give the demonstration here that, due to the existence of ample experimental data, we

have identified at least one nucleus where such an  $\alpha$ -particle condensed state exists and then we discuss the indications and the likelihood that such states very naturally also are present in other nuclei and that it may be a quite general phenomenon in nuclear systems.

The nucleus we want to draw our attention to is  $^{12}\text{C}$ . We, indeed, will give strong arguments that the  $0_2^+$  state at 7.654 MeV in  $^{12}\text{C}$  is a state of  $\alpha$ -particle condensate nature.

First, it should be noticed that the  $0_2^+$  state in  $^{12}\text{C}$  is actually, as  $^8\text{Be}$ , unstable and situated about 300 keV above the three  $\alpha$ -break up threshold. This state only is stabilised by the Coulomb barrier. It has a width of 8.7 eV and a corresponding lifetime of  $7.6 \times 10^{-17}$  s. As well known, this state is of extreme astrophysical importance concerning the synthesis of  $^{12}\text{C}$  in the universe and its existence was predicted in 1953 by the astrophysicist F. Hoyle [2] and shortly after discovered by W. A. Fowler in 1957 [3]. It is also well known that this Hoyle state, as it is called now, is a notoriously difficult state for any nuclear theory and for example the most modern no-core shell model calculations predict the  $0_2^+$  state in  $^{12}\text{C}$  to occur at around 17 MeV, that is more than two times its actual value [4]. This fact alone may tell us that the Hoyle state must have a very unusual structure and it is easy to understand that, should it indeed have a loosely bound three  $\alpha$ -particle structure, a shell model type of calculation would have great difficulties to reproduce its properties (Fig. 2). However, about 30 years ago, two Japanese physicists,

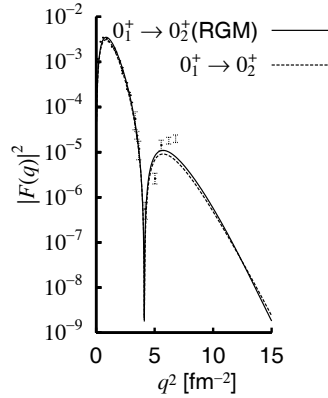


Fig. 2 – Experimental values of inelastic form factor in  $^{12}\text{C}$  to the Hoyle state [16] are compared with our values and those given by Kamimura *et al.* in [5] (RGM). In our result, the Hoyle state wave function is obtained by solving the Hill-Wheeler equation based on (2).

M. Kamimura [5] and K. Uegaki [6], with their collaborators, almost simultaneously achieved to reproduce the Hoyle state from a microscopic theory, i.e.

employing a twelve nucleon wavefunction together with a Hamiltonian containing an effective nucleon-nucleon interaction. Their works, at that time, probably did not attract the wide spread attention they deserved and the true importance of their achievement is eventually only recognised now. Both authors started from practically the same ansatz for the  $^{12}\text{C}$  wavefunction, having the following three  $\alpha$ -cluster structure:  $\langle \vec{r}_1 \dots \vec{r}_{12} | ^{12}\text{C} \rangle = \mathcal{A}[\chi(\vec{\mathcal{R}}, \vec{s})\phi_1\phi_2\phi_3]$  where  $\phi_i$  is an intrinsic  $\alpha$ -particle wavefunction of prescribed Gaussian form, i.e.  $\phi = \exp(-[(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_1 - \vec{r}_3)^2 + \dots]/b^2)$  where the size parameter is adjusted to get the rms value of the free  $\alpha$ -particle radius right.  $\chi(\vec{\mathcal{R}}, \vec{s})$  is the yet to be determined three body wavefunction for the center of mass motion of the three  $\alpha$ 's with  $\vec{\mathcal{R}}$  and  $\vec{s}$  the corresponding Jakobi coordinates. The unknown function  $\chi$  was then determined via a GCM [6] and RGM [5] calculation using the Volkov I and Volkov II nucleon-nucleon forces which well reproduce  $\alpha$ - $\alpha$  phase shifts. The precise solution of this complicated three body problem was, 30 years back, a truly pioneering work. The results were up to expectation. The position of the Hoyle state as well as other properties like inelastic form factor and transition probability successfully reproduced the experimental data. Other states of  $^{12}\text{C}$  below and around the energy of the Hoyle state were also successfully described. It also was already recognised that the three  $\alpha$ 's in the Hoyle state form sort of gas like state, a feature which had already been pointed out by H. Horiuchi [7] prior to the works of [5] and [6] using the orthogonality condition model (OCM) [8]. All of these authors also concluded from their studies that the linear chain state of three  $\alpha$ -particles, as this was postulated by Morinaga many years back [9], had to be excluded.

Though, as already said, the afore mentioned authors all had already stressed the somewhat  $\alpha$ -gas like nature of the Hoyle state, eventually two important aspects were missed at that time. First comes the fact that, because all three  $\alpha$ 's move in identical  $S$ -wave orbits, this forms an  $\alpha$ -condensate state, albeit not in the macroscopic sense. Second is that the complicated three body wave function can be replaced by a structurally and conceptually very simple microscopic three  $\alpha$  wave function of the condensate type which has practically 100 percent overlap with the previous ones [10]. We now shortly want to describe this condensate wave function.

For this we make an analogy to the Cooper pair BCS wave function of ordinary pairing. The latter wave function can be written in position space as

$$\langle \vec{r}_1 \dots \vec{r}_N | \text{BCS} \rangle = \mathcal{A}[\phi(\vec{r}_1, \vec{r}_2)\phi(\vec{r}_3, \vec{r}_4) \dots \phi(\vec{r}_{N-1}, \vec{r}_N)], \quad (1)$$

where  $\phi(\vec{r}_1, \vec{r}_2)$  is the Cooper pair wave function, including spin and isospin, which is being determined variationally by the well known BCS equations. As before  $\mathcal{A}$  is the antisymmetriser. The condensate character of the BCS ansatz is born out by the fact that we have a product of  $N/2$  times the same pair wave

function  $\phi$ . Formally it now is a simple matter to generalise (1) to  $\alpha$ -particle condensation. We write

$$\langle \vec{r}_1 \dots \vec{r}_N | \Phi_{n\alpha} \rangle = \mathcal{A}[\phi_\alpha(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \phi_\alpha(\vec{r}_5, \dots, \vec{r}_8) \dots \phi_\alpha(\vec{r}_{N-3}, \dots, \vec{r}_N)], \quad (2)$$

where  $\phi_\alpha$  is the wave function common to all condensed  $\alpha$ -particles. Of course, in general, the variational solution for  $\phi_\alpha(\vec{r}_1, \dots, \vec{r}_4)$  from (2) is extraordinarily more complicated than to find the Cooper pair wave function of (1). However, in the case of the  $\alpha$ -particle and for relatively light nuclei, the complexity of the problem can be reduced dramatically. This stems from the fact that, as was already recognised by the authors in [5, 6], an intrinsic wave function of the  $\alpha$ -particle of Gaussian form with only the size parameter  $b$  to be determined, is an excellent variational ansatz (see above). In addition, and here resides the essential and crucial novelty of our wave function, even the center of mass motion of the various  $\alpha$ -particles can very well be described by a Gaussian wave function with, this time, a size parameter  $B \gg b$  to account for the motion over the whole nucleus. We therefore write

$$\phi_\alpha(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = e^{-\vec{R}^2/B^2} \phi(\vec{r}_1 - \vec{r}_2, \vec{r}_1 - \vec{r}_3, \dots), \quad (3)$$

where  $\vec{R} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)/4$  is the c.m. coordinate of one  $\alpha$ -particle and  $\phi(\vec{r}_1 - \vec{r}_2, \dots)$  is the same intrinsic  $\alpha$ -particle wave function of Gaussian form as already used in [5, 6] and written out above. Of course, in (2) the center of mass  $\vec{X}_{cm}$  of the three  $\alpha$ 's, i.e. of the whole nucleus, should also be eliminated what is easily achieved by replacing  $\vec{R}$  by  $\vec{R} - \vec{X}_{cm}$  in each of the  $\alpha$  wave functions in (2). The  $\alpha$ -particle condensate wave function (2) with (3), proposed in [11] and henceforth called THSR-wavefunction, now depends only on two parameters,  $B$  and  $b$ . The expectation value of the microscopic Hamiltonian

$$\mathcal{H}(B, b) = \langle \Phi_{n\alpha}(B, b) | H | \Phi_{n\alpha}(B, b) \rangle / \langle \Phi_{n\alpha} | \Phi_{n\alpha} \rangle \quad (4)$$

can be evaluated and the corresponding two dimensional energy surface quantised in using the two parameters  $B$  and  $b$  as Hill-Wheeler coordinates.

Before coming to the results, let us discuss the THSR- wave function a little more. The innocuously looking ansatz (2) with (3) is actually more subtle as it might seem. One should realise that it contains two limits exactly: if  $B = b$ , then (2) boils down to a standard Slater determinant with harmonic oscillator wave functions with oscillator length  $b$  as the single variational parameter. This holds because (3), with  $B = b$ , becomes a product of four identical Gaussians and the antisymmetrisation creates all the necessary  $P$ ,  $D$ , etc. harmonic oscillator wave functions automatically [11]. On the contrary, when  $B \gg b$ , the density of  $\alpha$ -particles is very low and in the limit  $B \rightarrow \infty$ , the average distance between  $\alpha$ -particles is so large that the antisymmetrisation between  $\alpha$ 's can be neglected, i.e. the operator  $\mathcal{A}$  in front of (2)

can be taken off. Our wave function then becomes an ideal gas of independent condensed  $\alpha$ -particles, i.e. a pure product state of  $\alpha$ 's! On the other hand, in realistic cases, the antisymmetriser  $\mathcal{A}$  can not be neglected and the evaluation of the expectation values in (4) becomes an analytical (but non-trivial) task. For the Hamiltonian in (4) we took the one of [12] with an effective nucleon-nucleon force of the Gogny type whose parameters have been adjusted to  $\alpha$ - $\alpha$  scattering phase shifts about 15 years back. It also leads to very reasonable properties of nuclear matter. Our theory is therefore free of any adjustable parameter. The energy landscapes  $\mathcal{H}(B, b)$  for various  $n\alpha$  nuclei are interesting by themselves [13] but for brevity not shown here.

### 3. RESULTS FOR FINITE NUCLEI

As already mentioned above, the wave function constructed from the Hill-Wheeler equation based on (2), (3), (4), has practically 100 percent overlap with the ones in [5, 6], once the same Volkov force is used [10]. It is, therefore, not astonishing that we also get very similar results to theirs. For  $^{12}\text{C}$  we obtain two eigenvalues: the ground state and the Hoyle state. Theoretical values for positions, rms values, transition probabilities, compared to the data, are given in Table 1. From the comparison of the rms radii we see that the

Table 1

Comparison of the binding energies, rms radii ( $R_{\text{r.m.s.}}$ ), and monopole matrix elements ( $M(0_2^+ \rightarrow 0_1^+)$ ) for  $^{12}\text{C}$  given by solving Hill-Wheeler equation based on (2) and by RGM [5]. Volkov No. 2 force as the effective two-nucleon force is adopted in the two cases, for which the  $3\alpha$  threshold energy is calculated to be  $-82.04$  MeV

		condensate w.f. (H.W.)	RGM [5]	Exp.
$E$ (MeV)	$0_1^+$	-89.52	-89.4	-92.2
	$0_2^+$	-81.79	-81.7	-84.6
$R_{\text{r.m.s.}}$ (fm)	$0_1^+$	2.40	2.40	2.44
	$0_2^+$	3.83	3.47	
$M(0_2^+ \rightarrow 0_1^+)$ (fm <sup>2</sup> )		6.45	6.7	5.4

volume of the Hoyle state is a factor 3 to 4 larger than the one of the ground state of  $^{12}\text{C}$ . This is the aspect of dilute gas state we were talking about above. Constructing an  $\alpha$ -particle density matrix  $\rho(\vec{R}, \vec{R}')$ , in integrating out of the total density matrix all intrinsic  $\alpha$ -particle coordinates, we find in diagonalising this density matrix that the corresponding  $0S$   $\alpha$ -particle orbit is occupied to 70 percent by the three  $\alpha$ -particles [14, 15]. This is a huge percentage, underlining the almost ideal  $\alpha$ -particle condensate aspect of the Hoyle state. In this regard one should remember that superfluid  $^4\text{He}$  has only 10 percent of the particles

in the condensate! Let us also mention that for the ground state of  $^{12}\text{C}$  the  $\alpha$ -particle occupations are equally shared between  $0S$ ,  $0D$ , and  $0G$  orbits, thus invalidating a condensate picture for the ground state. Please notice that also the ground state energy of  $^{12}\text{C}$  is reasonably reproduced by our theory.

Let us now discuss the, to our mind, most convincing feature that our description of the Hoyle state is the correct one. As the authors of [5], we reproduce very accurately the inelastic form factor  $0_1^+ \rightarrow 0_2^+$  of  $^{12}\text{C}$ . This is shown in Fig. 3. The agreement with experiment is as such already quite im-

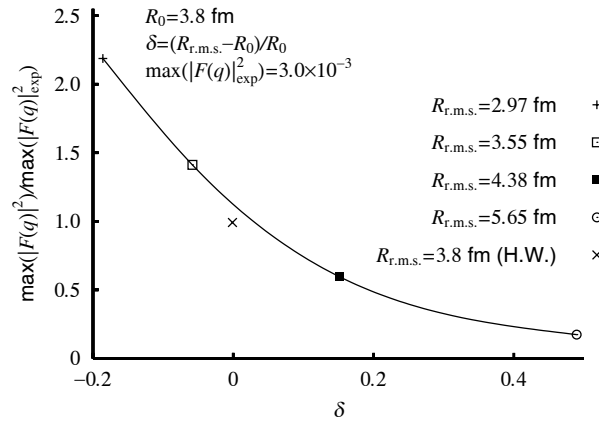


Fig. 3 – Present result of the inelastic form factor compared with experiment. RGM result corresponds to ref. [5] (left panel). The ratio of the value of maximum height, theory versus experiment, for the inelastic form factor, i.e.  $\max |F(q)|^2 / \max |F(q)|^2_{\text{exp}}$ , is plotted as a function of  $\delta$ , which is defined as  $\delta = (R_{\text{r.m.s.}} - R_0) / R_0$ .  $R_{\text{r.m.s.}}$  and  $R_0$  are the rms radii corresponding to the wave function (2) and the one obtained by solving the Hill-Wheeler equation based on (2), respectively (right panel).

pressive. Additionally, however, we made the following study shown in Fig. 3. We artificially varied the extension of the Hoyle state and studied the influence on the form factor. We found that the overall shape of the form factor only varies little, for instance in what concerns the position of the minimum. On the contrary, we found a strong dependence on the absolute magnitude of the form factor and in Fig. 3 we also plot the variation of the height of the first maximum of the inelastic form factor as a function of the percentage change of the rms radius of the Hoyle state [17]. It can be seen that a 20 percent increase of the rms radius decreases the maximum by a factor of two! This strong dependence of the magnitude of the form factor makes us firmly believe that the agreement with the actual measurement is practically a proof that our calculated wide extension of the Hoyle state corresponds to reality.

The Hoyle state can be considered as the ground state of the  $\alpha$ -particle condensate. Exciting one  $\alpha$ -particle out of the condensate and putting it into

the  $0D$  orbit reproduces the experimentally measured position of the  $2_2^+$  state in  $^{12}\text{C}$ . Without going into details, we also state that the width of this state is correctly reproduced [18]. It is tempting to imagine that the  $0_3^+$  state which experimentally is almost degenerate with the  $2_2^+$  state is obtained by lifting one  $\alpha$ -particle into the  $1S$  orbit. First theoretical studies [19] indicate that this view might indeed be true. However, its width ( $\sim 3\text{MeV}$ ) is very broad what makes a theoretical treatment rather delicate and further investigations are necessary to validate this picture. At any rate, it would be very satisfying, if the triplet of states, i.e.  $0_2^+, 2_2^+, 0_3^+$ , could all be explained from the  $\alpha$ -particle point of view, since those three states are precisely the ones which can not be explained within a (no core) shell model approach [4].

In conclusion, in what concerns  $^{12}\text{C}$ , we think we have accumulated enough facts to become convinced that the Hoyle state is, indeed, what one can call an  $\alpha$ -particle condensate state, being aware of the fact that ‘condensate’ for only three particles constitutes a certain abuse of the word. We, however, should remember in this context that also in the case of nuclear Cooper pairing, only a few Cooper pairs are sufficient to obtain clear signatures of superfluidity in nuclei!

What about  $\alpha$ -particle condensation in heavier nuclei? Of course, once one accepts the idea that the Hoyle state is essentially a state of three free  $\alpha$ -particles held together only by the Coulomb barrier, it is hard to believe that analogous states should not also exist in heavier  $n\alpha$  nuclei like  $^{16}\text{O}$ ,  $^{20}\text{Ne}$ ,  $^{24}\text{Mg}$ , . . . . At least our calculations systematically always show a  $0^+$ -state close to the  $\alpha$ -particle disintegration threshold. For example in  $^{16}\text{O}$  we obtain three  $0^+$ -states: the ground state at  $E_0 = -124.8\text{ MeV}$  (experimental value:  $-127.62\text{ MeV}$ ), a second state at excitation energy  $E_{0_2^+} = 8.8\text{ MeV}$  and a third one at  $0_3^+ = 14.1\text{ MeV}$ . The threshold in  $^{16}\text{O}$  is at  $14.4\text{ MeV}$ . Unfortunately the experimental situation in  $^{16}\text{O}$  is by far not so complete as the one in  $^{12}\text{C}$ . For example no transition probability measurement of  $0^+$ -states around threshold in  $^{16}\text{O}$  nor inelastic form factors do exist. Recently Wakasa [20] identified a new  $0^+$ -state at  $13.5\text{ MeV}$  in  $^{16}\text{O}$  which is the 5-th  $0^+$ -state. There are indications that it might be the  $\alpha$ -condensate state [21].

An interesting question is how many  $\alpha$ 's can maximally be in a self bound  $\alpha$ -gas state. For answering this question, a schematic investigation using an effective  $\alpha$ - $\alpha$  interaction in an  $\alpha$ -gas mean field calculation of the Gross-Pitaevsky type was performed. Our estimate yields [22] a maximum of ten  $\alpha$ -particles which can be held together in a condensate. However, a couple of extra neutrons may stabilise larger condensates.

Another interesting idea concerning  $\alpha$ -particle condensates was put forward by von Oertzen and collaborators [23, 24]. Adding more and more  $\alpha$ -particles to e.g. the  $^{40}\text{Ca}$  core, one sooner or later will arrive at the  $\alpha$ -particle

drip. Therefore it may need little further excitation energy to shake loose further  $\alpha$ -particles, so that an  $n\alpha$ -condensate could be created on top of an inert  $^{40}\text{Ca}$  core. Similar ideas also have been advanced by Ogloblin [25] who imagines a three  $\alpha$ -particle condensate on top of  $^{100}\text{Sn}$  and earlier by Brenner and Gridnev who think having detected gaseous  $\alpha$ -particles in  $^{28}\text{Si}$  and  $^{32}\text{S}$  on top of an inert  $^{16}\text{O}$  core [26].

#### 4. CONCLUSION

In conclusion, we see that the idea of  $\alpha$ -particle condensation in nuclei has already triggered a lot of new works and ideas inspite of the fact that so far strong identification of such a state only exists in  $^{12}\text{C}$ . However, the possibility of the existence of a completely new nuclear phase where  $\alpha$ -particles play the role of quasi-elementary constituents is surely fascinating and hopefully many more  $\alpha$ -particle states will be detected in the near future.

Let us end with some general remarks. Strongly bound  $\alpha$ -particles exist in nuclear physics because there are four different fermions, i.e. protons and neutrons with both spin up/down and roughly equal pairwise attraction among the four possibilities. One can also see it in a mean field picture where all four nucleons can occupy the lowest 0S-state whereas had we only e.g. neutrons, i.e. one species, two out of the four nucleons would have to be put in the next 0P-shell what is energetically very penalising. It is, therefore, conceivable that a similar situation with respect to quartetting could be created with fermions in traps, if one captured them in four different magnetic substates. Under the condition that they all attract them with more or less equal strength, one could create the very interesting situation of quartet condensation on a macroscopic scale. It would certainly be promising to investigate such experimental possibilities.

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