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REVIEW PAPERS

SIMPLICITY AND COMPLEXITY IN NUCLEAR STRUCTURE

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Abstract. Atomic nuclei are complex many-body quantum systems composed of two kinds of strongly interacting particles. Two great challenges of nuclear physics, which are shared in common with many other branches of modern science, are to understand how these complex systems can be constructed from simple ingredients, and how it is that the resulting systems can display such astonishing regularities and simplicities. The discussion here focuses on aspects of the latter, with particular emphasis on recent developments in the study of the evolution of nuclear structure as a function of the number of constituent protons and neutrons.

Key words: nuclear structure, simplicity and complexity, phase transitions, structural evolution, order and chaos.

1. INTRODUCTION: COMPLEXITY AND REGULARITY

Atomic nuclei are remarkable objects. They are composed of up to hundreds of nucleons, of two types, interacting with the strong and electromagnetic forces, all contained within a bound quantal system with a volume 10^{-10} of that of the atom, yet with 99.9% of the total atomic mass. They exist in the femto world, and are the only link between QCD and everything else, but they share much in common with their many-body analogues in the mesoscopic nano-scale world of atoms and biology, and they have profound implications for the giga-scale of the cosmos and for understanding the temporal history of the universe back to the first moments after the big bang. Some of these links with other areas of science are illustrated in Fig. 1 along with some themes in common with the nano- and giga-realms.

The study of atomic nuclei is also linked to the span of modern science by virtue of two great challenges facing the study of almost any many-body system. These are the themes and challenges of *complexity from simplicity* and *simplicity out of complexity.* These can be summarized by the following over-arching questions:

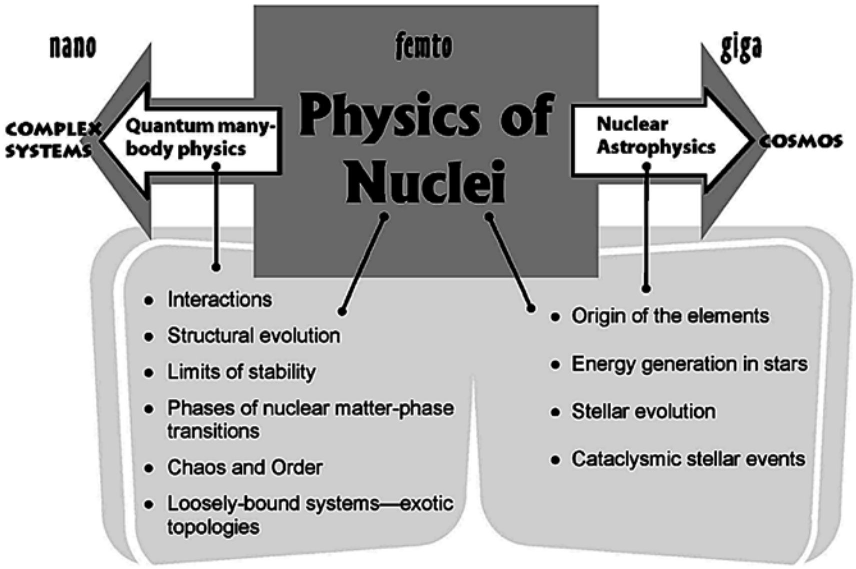

Fig. 1 – Relations of the physics of nuclei to other areas of modern science.

• *How can complex many-body systems be constructed from simple ingredients?*

In the nuclear case these ingredients are (at distances greater than a few tenths of a fermi): nucleons, the forces of nature, quantum mechanics, and a few Principles (Pauli, the Uncertainty Principle, conservation of angular momentum...)

• *How can these complex objects display such astonishing simplicities and regularities?*

In nuclei, these regularities are truly remarkable. Some are well understood, others partially so, while still other challenge the future. The focus in this paper will be on the regularities and simple behavior exhibited by atomic nuclei. It is worth starting with a few examples.

Although nuclei can be comprised of hundreds of nucleons, each with its own angular momentum, somehow it happens that all even-even nuclei have 0^+ ground states. Why is this? The origins lie in simple features of the fundamental short range character of the strong force (meson exchange under the time/distance constraints of the Uncertainty Principle) and in the Pauli Principle. Many even-even nuclei also exhibit energy ratios of the first two excited states, with angular momenta 2^+ and 4^+, of almost exactly 3 1/3. That we understand this as a reflection of the rotation of a deformed ellipsoid begs the question of how a nucleus with 100 or more nucleons can conspire to behave so simply. Indeed, other aspects of rotational motion are no less remarkable. The energy formula for a quantum axially symmetric top is $E(J) \sim J(J + 1)$. This implies that the *differences* in level energies are linear in J and that the second differences are constant. These second differences are simply the differences of γ-ray transition energies in a cascade of transitions down a rotational band in a deformed nucleus. Fig. 2 shows these differences for two of the best examples. They show complementary features of regularity in any physical system. The first is that such regularity almost always arises from a symmetry. In this case it is that of the group O(3). That this group structure is preserved, as in the upper plot in Fig. 2, over such a large range of angular momenta is striking evidence of the stability of the nuclear shape, despite disrupting centrifugal forces that become enormous at high spin. Perhaps even more surprising is the very fact that we can speak of a nuclear *shape* at all in a finite, discrete, microscopic quantal system. We will return to the concept of dynamical symmetries underlying such phenomena later.

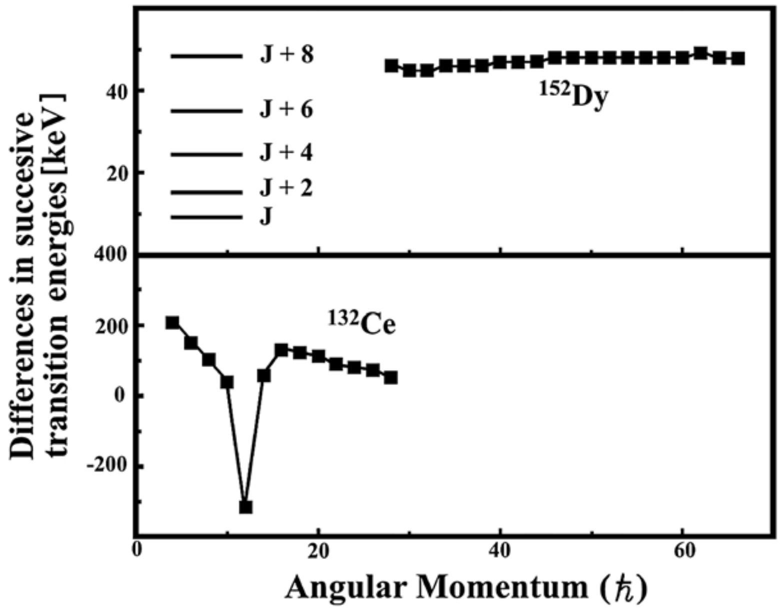

Fig. 2 – Differences of transition energies in two rotational bands in even-even deformed nuclei. (Figure courtesy of C.W. Beausang).

The second aspect of any regular feature of a system is that its regularities highlight and magnify deviations from them. The lower curve in Fig. 2 illustrates this with a sharp dip in an otherwise smooth systematic trend. The hitch can be explained in terms of the phenomenon of band crossing. This detail, though, is less important than the concept of using symmetries and regularities as a magnifying glass to see new physics.

Fig. 3 illustrates another characteristic feature of nuclear spectra that occurs sporadically but regularly in specific regions – these spectra are totally different than rotational ones in which the successive energy spacings *increase*. Here they

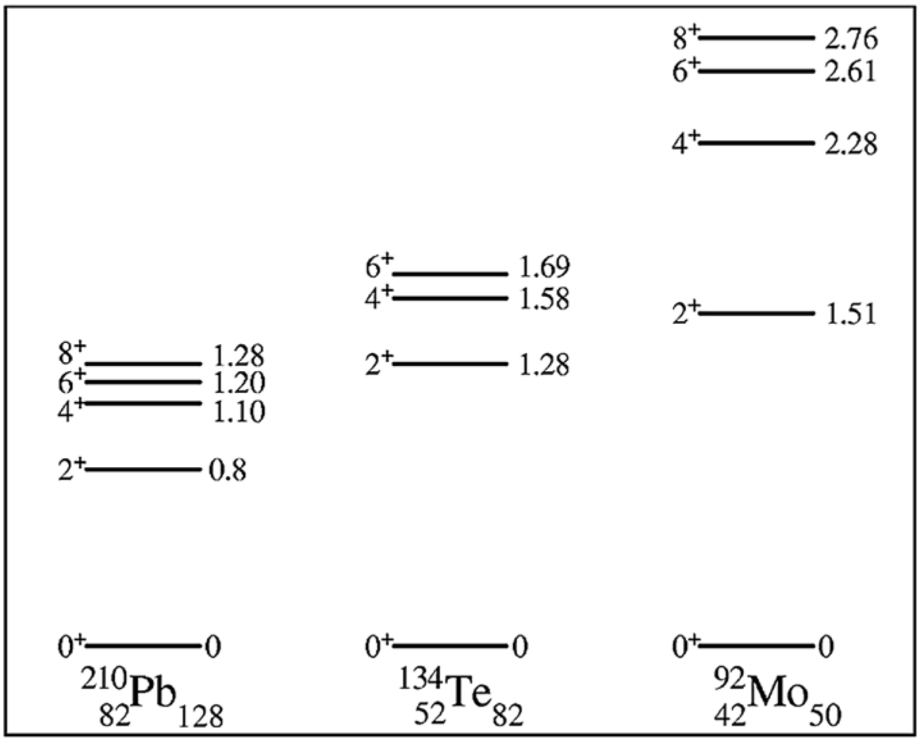

Fig. 3 – Typical patterns of yrast levels in medium mass and heavy nuclei with two particles outside a doubly magic core.

decrease. The figure shows almost identical looking spectra (except for scale) for nuclei in as diverse mass regions as $A \sim 90$, 130, and 210. Again, this can be understood in terms of short range residual interactions and the Pauli Principle, but the key point here is the multiple cloning of such spectra across the span of medium and heavy nuclei.

As a final example, regularity frequently occurs in nuclear systematics as a function of nucleon number. Consider the energy ratios $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ alluded to at the beginning. In deformed nuclei $R_{4/2}$ values are often within 1% of 3 1/3. In nuclei near closed shells they are less than 2 (see Fig. 3) while in moderately collective nuclei (vibrators) they have values near 2.0–2.2. Remarkably, the evolution of $R_{4/2}$ values between these limits is surprisingly smooth, as shown in Fig. 4 (left). The smoothness of this trajectory or of the correlation between $E(4_1^+)$ and $E(2_1^+)$ seen in Fig. 4 (right) are not understood. Nevertheless, the existence of the smoothness highlights deviant behavior and Fig. 4 (left) has a spectacular example of this, for two Hg nuclei. Here again, the point is the magnifying effect of regularity, not the specific details of a particular case but it is worth noting that this effect is understood to be due to mixing of “normal”

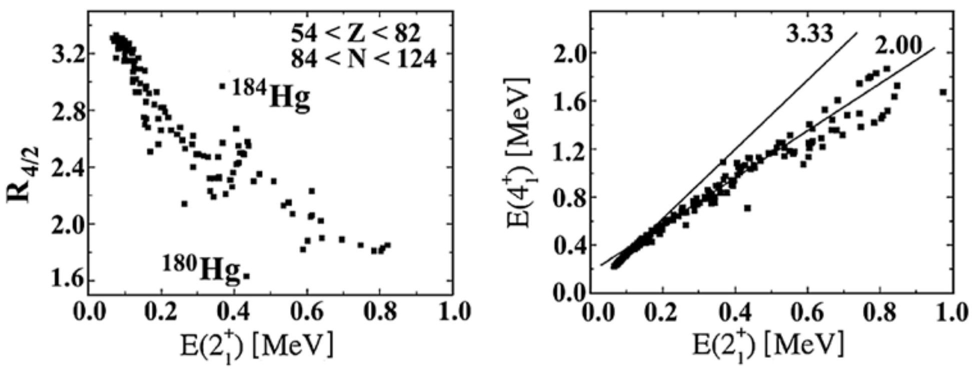

Fig. 4 – Left: $R_{4/2}$ values for nuclei in the rare earth region, showing a smooth evolution. Two anomalous nuclei are labelled. Right: The remarkable correlation of $E(4_1^+)$ against $E(2_1^+)$ for all collective ($R_{4/2}$ ~ 2.0) nuclei from Sr to Pb. The trajectory has two distinct segments with empirically fitted slopes that, incredibly, have the values 2.00 {1} and 3.33 {1} characteristic of the vibrator and rotor, respectively. The sharp kink at $E(2_1^+)$ ~ 120 keV is at the critical point of the spherical-deformed phase transition described by X(5). From ref. [1].

and cross-shell intruder states. Of course, the very concept of shells and cross-shell excitations itself is a simple manifestation of nuclear regularity, and the Pauli Principle, as it arises from the clustering and filling of single particle levels in a 1-body potential. Such clustering is naturally endemic to almost any plausible potential and arises from simple considerations of solutions to the Schrödinger equation in 3-dimensions. The mere fact that the thousands of 2-body nucleonic interactions in a nucleus can be reduced to and simulated by a 1-body potential (although, of course, only approximately) is itself one of the most remarkable simplicities of atomic nuclei.

2. SYMMETRIES AND PHASE TRANSITIONS

At this point, it is useful to formalize a bit our description of nuclei. We do this with the IBA model [2] though other collective models would suffice (albeit, generally, with more parameters).

We have spoken of symmetries. Dynamical symmetries of the nuclear Hamiltonian are an inherent feature of the IBA, whose U(6) group structure leads to sub-group chains denoted U(5), SU(3) and O(6), which describe vibrational, axially symmetric rotational, and γ-soft rotational nuclei, respectively. With three symmetries, it is natural to denote them with the device of a (symmetry) triangle, shown in Fig. 5. Typical partial level schemes of these symmetries are shown at their respective vertices. Most nuclei, of course, do not directly manifest these exact symmetries. Nevertheless, the symmetries provide benchmarks or paradigms of

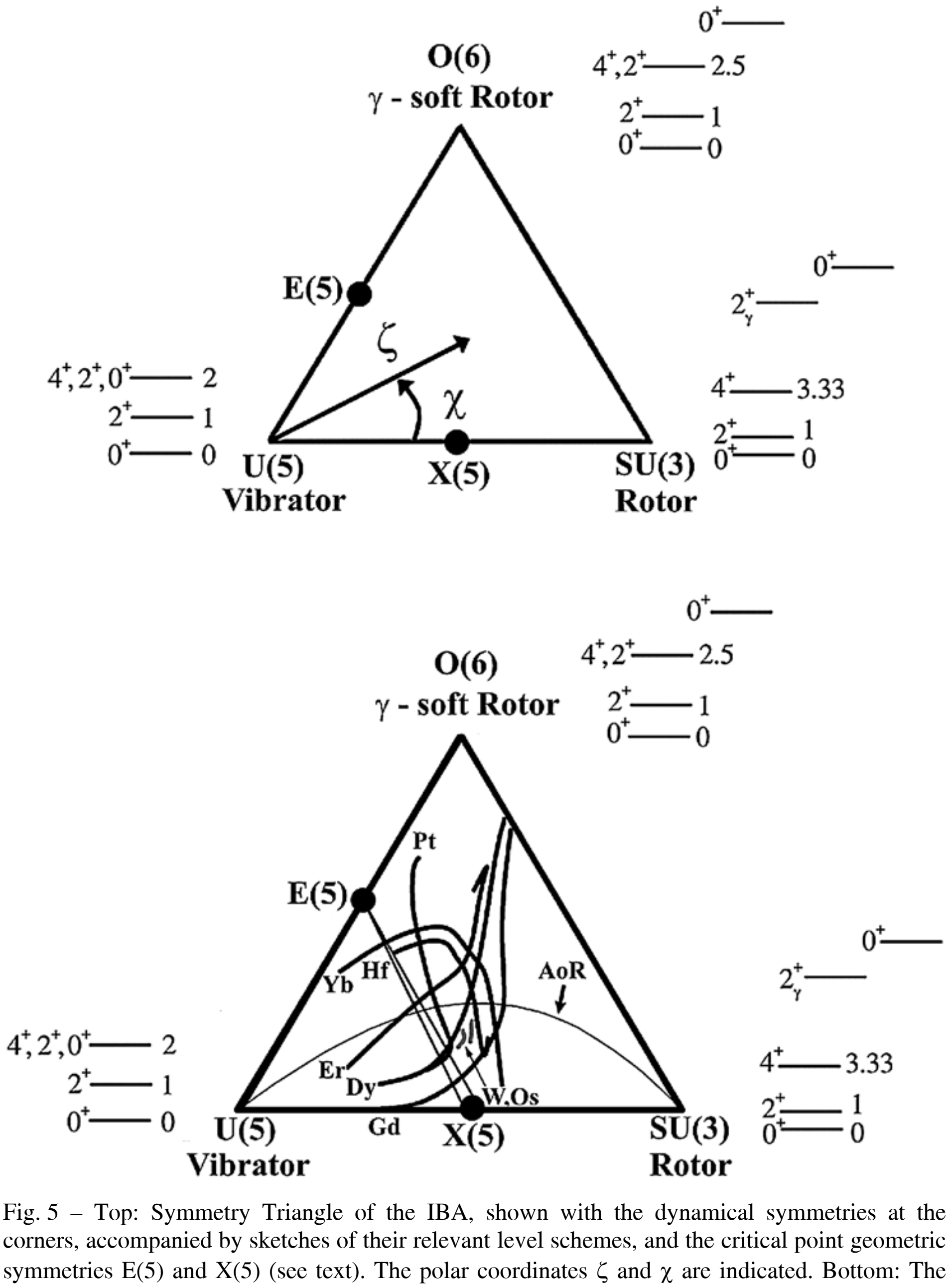

Fig. 5 – Top: Symmetry Triangle of the IBA, shown with the dynamical symmetries at the corners, accompanied by sketches of their relevant level schemes, and the critical point geometric symmetries E(5) and X(5) (see text). The polar coordinates ζ and χ are indicated. Bottom: The trajectories of nuclei from Gd-Pt are shown, along with the line of first order phase transition connecting X(5) to E(5) and the Arc of Regularity (AoR) [see text]. Based on refs. [8–10].

structure and allow for a simple *mapping* procedure to locate any collective nucleus in the triangle. The basic idea is embodied in an Ising-like Hamiltonian:

$$H = \varepsilon H_{sph} + \kappa H_{def} \quad (1)$$

Where H_{sph} represents the Hamiltonian of a higher symmetry system (*e.g.*, a spherical vibrator), with coupling constant ε, while H_{def} has the lower symmetry of a deformed field with coupling constant κ. The resultant structure of a system described by eq. (1) is determined solely by the ratio ε/κ. For ε/κ large the spherical solution dominates, while for $\varepsilon/\kappa \to 0$ the nucleus described by eq. (1) becomes deformed. The transition in shapes occurs at a critical value $(\varepsilon/\kappa)_{crit}$.

The simplest IBA Hamiltonian in the CQF [3] is of this type, namely

$$H = \varepsilon n_d - \kappa Q \cdot Q \quad (2)$$

With only the one parameter ε/κ, eqs. (1) or (2) have two limits only. The third dynamical symmetry of the IBA occurs because Q contains an additional internal parameter χ which determines the axial symmetry and its stiffness. With two parameters, ε/κ and χ, any point in the 2-dimensional symmetry triangle can be labeled. Typically, this is done in terms of "polar" coordinates, a radius vector ζ from the U(5) vertex that is related to ε/κ, and an angular parameter χ which takes the value $-\sqrt{7}/2$ along the horizontal axis and 0 along the γ-soft U(5) to O(6) leg. [An alternate parameterization using η instead of ζ is often encountered but the physics is the same.] The terms in eq. (2) depend on the boson number, N_B, defined as half the number of valence nucleons. Thus the structure resulting from a given (ζ, χ) parameter set is boson number dependent.

Observables such as $R_{4/2}$ vary systematically across the triangle. This is illustrated in Fig. 6 (top), with calculations for a constant boson number ($N_B = 10$). Fig. 6 (top) also shows an important feature of structural evolution in the IBA. Note that $R_{4/2}$ varies very slowly when moving away from the U(5) and SU(3) limits and that most of its change from 2.00 to 3.33 occurs in a narrow central region of the triangle, signaling an especially rapid structural change.

The sudden change in $R_{4/2}$ has recently been described in terms of phase transitional behavior, leading to a new class of *critical point symmetries* [4, 5] that describe nuclei *at* the phase transitional point. These are analytic, parameter-free (except for scale), symmetries of the geometric Bohr Hamiltonian and are denoted E(5) (for a second order vibrator to γ-soft rotor phase transition) and X(5) (for a first order vibrator to axial rotor phase transition). Remarkably, recent studies [6, 7] have provided striking empirical evidence for E(5) and, especially, for X(5). Together, the five symmetries labeled in Fig. 5 provide descriptions of the three idealized limits of nuclear structure (for cases where the proton and neutron fluids act in concert – as happens in nearly all even-even nuclei at low energy) and the two descriptions of phase transitional behavior between these symmetries. (Though

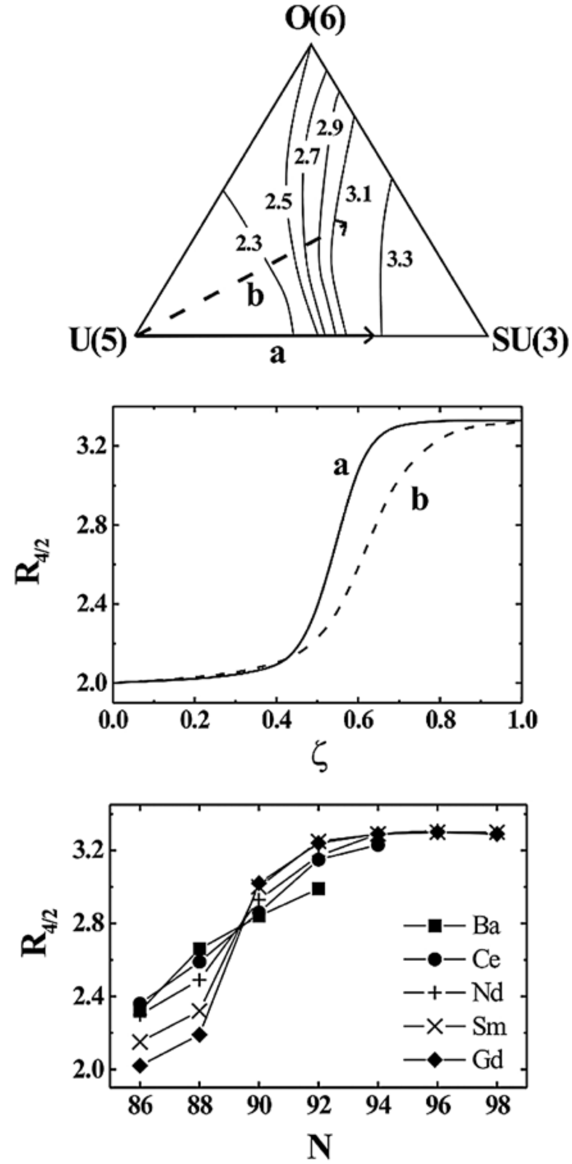

Fig. 6 – Top: Contours of constant $R_{4/2}$ calculated with IBA for boson number N_B = 10. Middle: $R_{4/2}$ values for trajectories "a" and "b". Note that since these trajectories are calculated for constant boson number they are schematic only and cannot be expected to precisely reflect actual systematics. Bottom: Experimental $R_{4/2}$ values in the A ~ 150 phase transitional region.

these latter are *not* symmetries of the IBA, we include them in Fig. 5 for pedagogical purposes.)

Fig. 6 (bottom) illustrates the observed behavior of $R_{4/2}$ in the Ba-Gd nuclei. Here we see, first of all, a rapid change as these nuclei go from spherical to deformed at $N \sim 90$. Secondly, the rapidity of the shape/phase transition as a function of neutron number varies from very rapid in Sm and Gd to rather smooth or gradual in Ba. Although other data is needed to project the location of a nucleus in the symmetry triangle, Fig. 6 suggests that Sm and Gd lie close to the bottom axis, passing through X(5), while the Ba and Ce trajectories are internal to the triangle. This is suggested by the two calculated trends in $R_{4/2}$ shown in Fig. 6 (labelled "a" and "b" in Fig. 6, middle) which mimic the behavior of the Sm-Gd and Ba-Ce trajectories, respectively.

The preceding discussion introduces us to the idea of mapping nuclei onto the triangle. This has recently been systematically done [8–10] for an extensive set of nuclei from Gd-Pt. The fits are excellent and are shown in some detail in the references. In all, 50–60 nuclei were fit. Most of these, except for the Pt isotopes, are for N = 104. They are illustrative of the mapping that can be done. These trajectories are shown in Fig. 5 (bottom) along with an unexpected discovery (see below) resulting from this systematic approach to structural evolution. First, though, we note that, while these trajectories are individually rather smooth, taken together they manage to occupy much of the triangle, pointing to the incredible diversity and variety of nuclear types.

In. Fig. 5 (bottom) we have sketched a narrow region connecting E(5) and X(5). Both an analysis with the coherent state formalism [11] of the IBA and with Landau theory [12] show that this is a line of first order phase transition that culminates at the second order E(5) point. Points along the central path in this region are not conceptually different than X(5), although their spectra differ in detail; however, X(5) differs in practice in that its solution are analytic. An important feature of the Landau analysis is that the triangle in thus divided into 2 phases (3 if oblate shapes are included), namely, spherical and deformed. It is topologically impossible to go from one phase to the other without therefore crossing a first order phase transition [except at the singular point E(5)]. And, indeed, the empirically-based trajectories shown in Fig. 5 (bottom) cross this line or come very close to it.

There is another fascinating upshot of structural evolution in the triangle. One will notice a curve looping through the triangle in Fig. 5 (bottom) from U(5) to SU(3). To understand this curve we return to the concept of dynamical symmetries and their group theoretical description. These symmetries can be expressed in terms of group chains, of the form U(1) ⊃ U(2) ⊃ U(3) ⊃ . . . O(3), ending in the group O(3), of rotational invariance. Each of these subgroups contains a number of "representations". Each representation is characterized by a particular value(s) of a characteristic quantum number(s). Each level is described (labeled) by a set of quantum numbers, one (or more) for each sub-group. The details and mathematical formulation of this are not germane to the present discussion. What is relevant is the existence of these "good" quantum numbers. They characterize the eigenvalue equation for the levels of each symmetry and these levels follow certain characteristic patterns (the constancy of transition energies cited earlier (see Fig. 2) is a typical characteristic of the SU(3) pattern, for example). As one moves away from the symmetries (except along the U(5)-O(6) leg) the states of the symmetries mix, and are no longer generally characterized by good quantum numbers. In the jargon of the IBA, the value of n_d is no longer a good quantum number away from U(5), τ is no longer good away from the U(5)-O(6) leg and λ, μ and K are no longer good away from SU(3). Regular patterns of level energies and transition rates change and, in general, colloquially speaking, the ordered character of the symmetries is replaced by disordered, or mixed, and chaotic spectra. This happens generally as one moves into the interior of the triangle.

However, a surprise was discovered about 15 years ago when it was found [13] that there is an arc cutting through the interior of the triangle along which regularity or order was largely reestablished. This is the Arc of Regularity shown in Fig. 5 (bottom). The absence of chaos almost certainly points to the emerging validity of some quantum number. In the case of the Arc of Regularity, the nature of this quantum number or what it labels are not yet known. Nevertheless, there must be something special about this locus. Indeed, there is, as it was recently pointed out [14] that it is the *unique* locus in the entire triangle along which the first excited 0^+ state and the second excited 2^+ state are nearly degenerate.

Interest in this Arc has re-emerged with the new mapping [8–10] which revealed [14] that a few nuclei (12 in particular) lie very close to the Arc. The fact that actual nuclei lie along the Arc of Regularity was unrecognized and unsuspected prior to this mapping. The fact that neighboring nuclei return to nearly chaotic regions is another fascinating aspect of structural evolution in this region that is yet to be understood. The discovery of nuclei lying along the Arc of Regularity is just one more example of the richness of nuclear structure and structural evolution and of the usefulness of a symmetry-based approach to structure.

To summarize this part of the discussion, the IBA provides a framework for understanding the wide variety of nuclear structure. Its dynamical *symmetries* reflect some of the remarkable regularities exhibited by *specific* nuclei. The *trajectories* of structural evolution and the concept of critical point symmetries account for some of the remarkable regularities, as well as sudden changes, in properties exhibited by *sequences* of nuclei. Mapping these trajectories can reveal new physics, such as the Arc of Regularity.

Of course, although collective modes, symmetries, and phase transitions are some of the most striking phenomena manifested by nuclei, they are only one side of the structural coin – the simplicity side. They describe nuclei in a *macroscopic* way. However, these symmetries appear, and structural evolution happens the way it does, because of the *microscopic* interactions of the individual nucleons, usually described in terms of effective residual interactions beyond the mean field potential. It is a critical challenge of microscopic theory to understand why these symmetries appear, where they appear, how structure evolves, and which new kinds of structure will appear in the great new frontiers of exotic nuclei. Microscopic theory thus reflects the complementary side of the many-body coin –

the challenge to build up complex systems and understand their behavior and the regularities they exhibit in terms of their constituent ingredients and the forces and physical laws they obey.

With our growing access to myriad new and exotic nuclei these two challenges are intertwined in new and fascinating ways. For example, what symmetries will appear if the mean field itself changes in loosely bound systems? What new phases will appear if proton and neutron fluids become decoupled in very neutron rich nuclei? With technological advances allowing access to such nuclei and their study with highly sensitive new generations of instruments, the twin approaches of microscopic analysis and macroscopic symmetry promise a future for understanding nuclear structure that looks brighter and more exciting than ever.

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