

## ELEMENTARY PARTICLES IN CURVED SPACES

I. Lazanu

University of Bucharest  
Faculty of Physics, Department of Atomic and Nuclear Physics  
POBox MG-11, Bucharest-Magurele, Romania  
e-mail: [I.Lazanu@yahoo.co.uk](mailto:I.Lazanu@yahoo.co.uk)

(Received July 22, 2004)

*Abstract.* Currently, in particle physics the phenomena are studied in flat space – time, starting from Poincaré's group. In fact, the existence of a non zero value of the stress-energy tensor in a region of the space produces a deformation of the space – time, as a gravitation effect, and thus it is necessary to discuss the physical implications if the phenomena are produced in curved spaces.

In this contribution, starting from symmetry considerations, some possible phenomenological consequences when the phenomena are studied in the particular case of de Sitter space are discussed.

*Keywords:* elementary particles, curved spaces, Poincaré group, de Sitter group, symmetries, invariances

PACS: 11.30.-j      Symmetry and conservation laws  
04.50.+h      Gravity in more than four dimensions  
11.30.Cp      Lorentz and Poincare invariance  
11.30.Er      Charge conjugation, parity, time reversal and other discrete symmetries

### INTRODUCTION

The traditional view of particle physics holds that the microscopic world can be regarded as an isolated system and forms a self-contained unit. In fact, the microscopic particle physics depends on the coherent properties of the macroscopic world, the environments. These characteristics are represented by the physical vacuum state. In the static situations the vacuum properties are averaged values, but each of these vacuum averages appears as an independent parameter. In accord with general relativity a density of energy in a region of the space-time is a source of gravitational field. Because the weak contribution of gravitational field, their contribution is generally neglected in particle physics, but in some situation this contribution

could be important and its effect is necessary to be estimated. Thus it is necessary to examine smaller spatial regions around the interaction vertex, or the situations when the fluctuations in the behaviour of the vacuum are relevant.

The easiest way to introduce classically the gravitation is to consider that gravity could be treated just as another field, and thus the idea is to develop a theory of a curved space-time. Feza Gürsey [1], Christian Fronsdal [2], W. Fushchych [3] et. al. have pioneering papers in this subject. Thus it is possible to put in evidence the differences in respect with the current formalism developed in a flat space-time. The way used in the present paper to attain this scope is the use the laws of symmetry. But, in a fundamental paper, E. Wigner [4] learned that: *"The approximate validity of laws of symmetry is, therefore, a very general phenomenon – it may be the general phenomenon;”all symmetry properties are only approximate”*.

In particle physics the theories are currently developed in Minkowski space-time starting from the Poincaré group.

A space of constant curvature has a group of motion – de Sitter that, though it differs from that of a flat space, has the same number of parameters and can permit some generalisations. A space of constant curvature may be realised as a pseudosphere in five dimensional spaces, and the group of motion is the set of pseudorotations that transforms this four-dimensional surface into itself. Thus when the radius of the sphere goes to infinity, the curvature tends to zero and we refund the Minkowski space and the group becomes the Poincaré group.

In this contribution only some aspects related to mass and discrete symmetries are discussed.

### de Sitter space - some group elements

In the following short review about the main elements of the de Sitter group and in the discussion of their physical consequences, we will use Gürsey's notations and arguments [1].

A four dimensional Riemannian space may admit a continuous group of motion with up to ten essential parameters. The maximum number of parameters is realised only for a space of constant curvature. De Sitter group is a group of transformations which defines a four-dimensional curved space which can be embedded in a five-dimensional flat space with coordinates  $\xi_i$  such that

$$\xi_a \xi_a = \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \xi_5^2 = R^2 \quad (1)$$

The five coordinates  $\xi_a$  and the four coordinates  $x_\mu$  are connected by the formulae:

$$\xi_\mu = \Phi(\sigma^2) x_\mu \quad (2a)$$

$$\xi_5 = R\Phi(\sigma^2) \left( 1 - \frac{\sigma^2}{4R^2} \right) \quad (2b)$$

or equivalently:

$$x_\mu = \frac{2\xi_\mu}{1 + \frac{\xi_5}{R}} \quad (3a)$$

$$\frac{\sigma^2}{4R^2} = \frac{1 - \frac{\xi_5}{R}}{1 + \frac{\xi_5}{R}} \quad (3b)$$

where  $\Phi(\sigma^2) = \left( 1 + \frac{\sigma^2}{4R^2} \right)^{-1} = \frac{1}{2} \left( 1 + \frac{\xi_5}{R} \right)$ ,  $R$  being the radius of curvature of the "universe" and

$$\sigma^2 = x_\mu x_\mu = x_1^2 + x_2^2 + x_3^2 - x_4^2 = r^2 - c^2 t^2.$$

Because  $\xi_4$  is pure imaginary, thus ratio  $\frac{\xi_5}{R}$  is real and are possible two cases both  $\xi_5$  and  $R$  are real or both imaginary, defining de Sitter universe (characterised by positive curvature) and anti de Sitter with negative spatial curvature, anti de-Sitter universe.

The Poincaré group is characterised by ten parameters that are fundamental quantities. Each of them is associated with a type of infinitesimal transformation. They determine how all-dynamical variables are affected by a change in the coordinate system. The generators are  $J_{\mu\nu}$  and  $P_\lambda$ .

De-Sitter group is characterised also by ten parameters. The homogeneous Lorentz group (depending on six parameters) is a subgroup of the de Sitter group. The remaining four transformations of the group rotate  $\xi_5$  into  $\xi_\mu$ , and thus induce non-linear transformations of  $x_\mu$ .

The generators of the de Sitter group are the rotation operators  $J_{ab}$ ,  $a, b = 1 \div 5$  in  $E_5$ .

Putting

$$\Pi_\mu = \frac{1}{R} J_{5\mu} \quad (5)$$

They obey the commutation relations

$$-\frac{i}{\hbar} [J_{\kappa\lambda}, J_{\mu\lambda}] = \delta_{\kappa\lambda} J_{\lambda\mu} - \delta_{\kappa\mu} J_{\lambda\nu} + \delta_{\lambda\mu} J_{\kappa\nu} - \delta_{\lambda\nu} J_{\kappa\mu} \quad (6a)$$

$$-\frac{i}{\hbar} [\Pi_\lambda, J_{\mu\nu}] = \delta_{\lambda\mu} \Pi_\nu - \delta_{\lambda\nu} \Pi_\mu \quad (6b)$$

$$-\frac{i}{\hbar} [\Pi_\mu, \Pi_\nu] = -\frac{1}{R^2} J_{\mu\nu} \quad (6c)$$

The first two relations are identical with those of the Poincaré group. In the zero curvature limit  $\lim_{R \rightarrow \infty} \Pi_\mu = P_\mu$  and thus  $[P_\mu, P_\nu] = 0$ , refunding the commutation relation characteristic for the Poincaré group. De Sitter group unites the linear momentum four-vector and the angular momentum six-vector into one geometrical entity.

For a free particle with mass  $m$ , the classical angular momentum 10-vector associated with the de Sitter group is defined as:

$$\lambda_{ab} = m \left( \xi_a \frac{d\xi_b}{ds} - \xi_b \frac{d\xi_a}{ds} \right) = const \quad (7)$$

generalising this way the current conservation laws of momentum and angular momentum.

The usual observables are:

$$\lambda_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu \quad (8a)$$

$$\pi_\mu = \frac{1}{R} \lambda_{5\mu} = \frac{m}{R} \left( \xi_5 \frac{d\xi_\mu}{ds} - \xi_\mu \frac{d\xi_{5a}}{ds} \right) = \left( 1 - \frac{\sigma^2}{4R^2} \right) p_\mu + \frac{x_\mu x_\nu}{2R^2} p_\nu \quad (8b)$$

In the limit of flat space the equation (8b) becomes in usual form:  $\pi_\mu \xrightarrow{R \rightarrow \infty} p_\mu = m \frac{dx_\mu}{d\tau}$ .

**The mass, the vacuum and its contribution to mass**

In the Poincaré group, there exist two quadratic invariant (Casimir) operators, which commute with all its generators. These are

$$I_1 = P_\mu P^\mu = P^2 \quad (9a)$$

and

$$I_2 = W_\mu W^\mu = W^2 \quad (9b)$$

where  $W^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} J_{\nu\rho} P_\sigma$  and  $\varepsilon^{\mu\nu\rho\sigma}$  is a metric tensor in four upper indices.

The eigenvalue of the first Casimir invariant operator is associated with the rest mass and consequently it is an invariant.

$$I_1 \rightarrow m^2 \quad (10a)$$

Thus, the concept of mass has its origin in the constancy of speed of light in all inertial referentials [5]. The second operator has an eigenvalue associated with (mass x spin)<sup>2</sup> of the system.

$$I_2 \rightarrow -m^2 s(s+1) \quad (10b)$$

In the de-Sitter group the first invariant operator becomes:

$$I_1 = -\frac{1}{2R^2} J_{ab} J_{ab} = -\Pi_\mu \Pi_\mu - \frac{1}{2R^2} J_{\mu\nu} J_{\mu\nu} \quad (11a)$$

with the eigenvalue:

$$I_1 = M^2 = m^2 - \frac{1}{2R^2} \left( \delta_{\kappa\nu} + \frac{x_\nu x_\kappa}{2R^2} \right) \lambda_{\mu\nu} \lambda_{\mu\kappa} \approx m^2 - \frac{\lambda_{\rho\kappa} \lambda_{\rho\kappa}}{2R^2}, \quad (11b)$$

so that the de Sitter invariant rest mass ( $M$ ) is a combination of the Poincaré rest mass and the angular momentum of a particle. Evidently, for a flat space,  $R \rightarrow \infty$ , and thus,  $M^2 \rightarrow m^2$ . In the concrete equation associated to Casimir operator, the second term is associated with the presence of gravity. In the de Sitter space, the significance of the second Casimir invariant operator is not completely established.

The environment effect is associated with the vacuum. In Minkowski space-time the vacuum state is build into QFT formalism supposing that it is invariant under Poincaré group. In this case the acceleration creates particles, phenomenon known as Unruh-Davies effect; so the true vacuum state is correctly determined only for inertial (non-accelerated) observers. The vacuum is not invariant under time translations. Poincaré invariance is not symmetry fulfilled by the vacuum state (of the various quantum fields) in the actual universe. This conclusion is obtained if the vacuum state is implemented in the cosmology: in universe, the vacuum energy is envisaged to change during phase transitions, so the vacuum is clearly not invariant under time translations.

The gravitational field being the source of curvature in GR; also, it is expected that this field will produce particles, and thus obscuring the concept of vacuum as a state with no particles.

The cosmological constant  $\Lambda$ , is a dimensional parameter with units of  $(\text{length})^{-2}$ . From the point of view of classical general relativity, there is no preferred choice for what the length scale defined by  $\Lambda$  might be.

Einstein determined the cosmological constant by the criterion that the equations of general relativity should correspond to Newtonian theory in the limit for weak gravitational fields and small velocities. At the present time, from galactic observations and in Solar system, the upper bound on the  $\Lambda$  – term is  $|\Lambda \leq 10^{-56} \text{ cm}^{-2}|$ .

Particle physics, however, brings a different perspective to the problem. The cosmological constant turns out to be a measure of the energy density of the vacuum – the state of lowest energy and thus could give a link between quantum vacuum energy and cosmological constant that permits to obtain some numerical evaluations. Thus,

$$\rho_{vac} \equiv \rho_{\Lambda} = \frac{\Lambda}{8\pi G} \quad (12a)$$

and this equivalence is the origin of the identification of the cosmological constant with the energy of the vacuum. In what follows, I will use the terms “vacuum energy” and “cosmological constant” essentially interchangeably. If this bound is interpreted as a bound on the vacuum energy density in QFT, it corresponds to

$$|\rho_{vac}| < 10^{-29} \text{ g/cm}^3 \propto 10^{-47} \text{ GeV}^4 \propto 10^{-9} \text{ erg/cm}^3.$$

in the actual Universe.

By contrast, theoretical estimates of various contributions to the vacuum energy density in QFT exceed the observational bound by at least 40 orders of magnitude. This large discrepancy constitutes the cosmological constant problem. Until now, an accepted explanation does not exist [6], [7], [8], [9], [10], [11]. Bludman and Ruderman [12] suggest that the small or zero value observed for the cosmological constant can be the

consequence of some supersymmetry or new gauge-invariance or may simply be another fundamental constant.

If  $E$  is an energy scale of a virtual process corresponding to a length scale  $l = \hbar c / E$ , then the gravitational self-energy density will be of the order of:

$$\rho = \frac{G \left( \frac{E}{c^2} \right)^2}{l} l^{-3} = \frac{GE^6}{c^8 \hbar^4} = \frac{E^6}{E_{pl}^2} \cdot \frac{1}{(\hbar c)^3} \quad \left[ \frac{\text{erg}}{\text{cm}^3} \right] \quad (12b)$$

If the requirements of spherical symmetry, dominant energy condition for a source term, regularity of density and finiteness of mass are realised [13], thus the metric of de Sitter space is:

$$ds^2 = \left( 1 - \frac{r^2}{R^2} \right) c^2 dt^2 - \left( 1 - \frac{r^2}{R^2} \right)^{-1} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\varphi^2 \quad (13)$$

$$\text{with } R^2 = \frac{3c^2}{8\pi G\rho}$$

In origin, at  $r = 0$ , the stress-energy tensor is  $T_{\eta\nu} = \rho c^2 g_{\mu\nu}$ . In this model  $\rho$  is the mass density at the origin and shell is identified with energy scale of the process. In accord with Einstein [14] we can interpret an elementary particle as the regular solution of nonlinear field equations located in the confined region where field tension and energy are particularly high.

Mansouri [15] has generalised Einstein's mass-energy relation for de Sitter space considering also a non-vanishing vacuum energy. In these hypotheses, the rest mass of a particle in de Sitter space ( $M$ ) becomes:

$$M^2 = m^2 - 3 \frac{mE_{vac}}{c^2} + \frac{E_{vac}^2}{c^4} (2 + s - s^2) \quad (14)$$

where  $m$  is its rest mass in Minkowski space and  $s$  is the spin.

If the current theories of particle physics based on spontaneously broken symmetries and phase transitions are correct, thus different energies scales will produce relevant modifications of space properties. For energies of the order of 200 GeV, corresponding to the electroweak phase transition, the vacuum energy is about  $E_{vac}^{EW} \propto 10^{-4} eV$  and the vacuum energy could attain a value  $E_{vac}^{20TeV} \propto 1eV$  at 20 TeV. Its effect must be relevant in the immediate neighbourhood of the interaction vertex, especially for light particles, as the neutrino for example.

### Discrete symmetries

In de Sitter space all knew discrete symmetries, associated with P, T and C inversions exist. Due to the existence of a supplementary dimension, consequence of the curvature and implicitly of the existence of gravity, a new discrete symmetry could be defined, which consists in the transformation:

$$(\vartheta): \quad \xi'_a = -\xi_a, \quad (15)$$

where  $a = 1 \div 5$ .

Thus, the inversions could be explicitly written as:

$$\vartheta: \quad x'_\mu = \frac{-2\xi_\mu}{1 - \frac{\xi_5}{R}} = -x_\mu \frac{4R^2}{\sigma^2} \quad (16a)$$

$$\frac{\sigma'^2}{4R^2} = \frac{4R^2}{\sigma^2} \quad (16b)$$

This inversion consists essentially in associating to each point another point on the opposite site on the hypersphere. The number of particles is doubled, the symmetry being the consequence of the presence of gravitational field. How can be interpreted the existence of this supplementary symmetry?

We suggest that this second set of particles could be associated with the existence of (predicted) mirror particles.

**a.** In the present theories, the symmetry of the Nature is broken. This theoretical edifice cannot be considered complete. Analysing this situation, T. D. Lee commented: *“All present theories are based on symmetry, but some symmetry quantum numbers are not conserved, suggesting that missing symmetries exist.”* [16].

E. Majorana proposed first the idea of symmetry of the Nature. Lee and Yang, in the last two paragraphs of their paper about mirror reflection [17], originally discussed the existence of mirror matter as an alternative symmetry possibility. They suggested that the transformation in the particle space corresponding to space inversion  $\vec{x} \rightarrow -\vec{x}$  should not be the usual transformation  $P$ , but  $PR$ , where  $R$  corresponds to the transformation of a particle into a reflected state in the mirror particle space. Later, after the observation of parity nonconservation, Landau assumed that  $R \equiv C$ , i.e. he suggested the identification of antiparticles with mirror matter. The idea of existence of mirror matter has later been discussed by Salam,

[18], Kobzarev, Okun and Pomenranchuk [19], Glashow [20], Pavšič [[21] and others which extended and gave mathematical support to this idea. In the last years Foot, Volkas and co-workers developed the model of symmetry of parity in more publications, see for example [22], [23], [24].

Initially, the concept of symmetry has only a geometrical significance. Matej Pavšič [21] extended the concepts of symmetry and mirror particles. He introduced the idea that the particles are not geometrical points, they are real physical objects, and thus they must have an internal structure which must be considered and which could have some effects on the global space time behaviour of the particles. The physical system is usually composed of sub-systems, at different levels of complexity and thus, it is necessary to consider also internal symmetry.

Classically, the so-called internal symmetries give concrete forms of the conservation laws and the dynamics of particles in terms of apparently non-kinematical properties of symmetries. In fact, if we consider that the system (particle) has an internal complexity, the concept of internal symmetry must be generalised, in the sense that it is necessary to consider non-kinematical properties, as additive quantum numbers generically named ( $\alpha$ ), but, also, it is necessary to introduce an internal space-time coordinates  $i \equiv i(\chi, \tau)$ .

**b.** If for internal space coordinates the physical significance is clear, what is the significance of the internal time variable? Could this variable be different for different physical systems and different behaviours in respect to time inversion?

For instance, in the description of an elementary particle it is necessary to consider that the state vector must be expressed in a more general form, as:  $\Phi(\vec{r}, t, s, i(\chi, \tau), \alpha)$  for example.

Thus, spatial and temporal inversion operators are introduced, corresponding to internal and external inversions. They transform the state vector of the system in the following way:

$$P_I : i(\chi, \tau) \rightarrow i'(\chi', \tau) = i(-\chi, \tau) \quad (17a)$$

$$T_I : i(\chi, \tau) \rightarrow i'(\chi, \tau') = i(\chi, -\tau) \quad (17b)$$

$$P_E : \Phi(\vec{r}, t, i, \alpha) \rightarrow \Phi'(\vec{r}', t, i, \alpha) = \Phi(-\vec{r}, t, i, \alpha) \quad (18a)$$

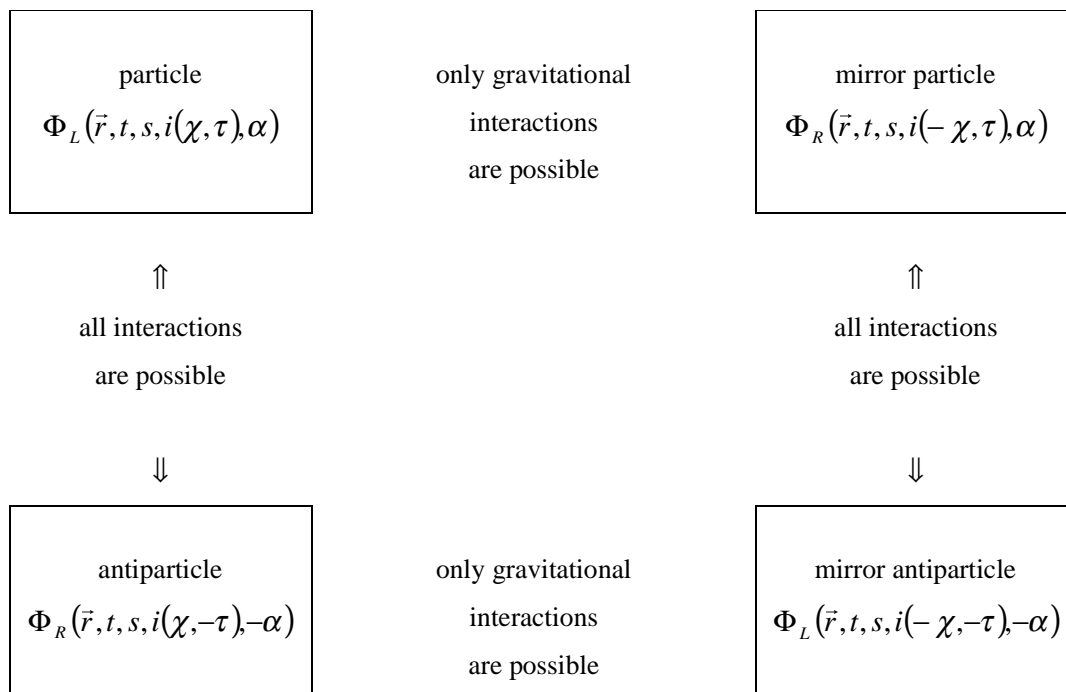
$$T_E : \Phi(\vec{r}, t, i, \alpha) \rightarrow \Phi'(\vec{r}, t', i, \alpha) = \Phi(\vec{r}, -t, i, \alpha) \quad (18b)$$

The main difficulty is related to the significance of internal time inversion, because in quantum theory the internal time variable isn't used.

To understand the problem that results from internal time inversion, a possible way is represented by Stueckelberg's relativistic quantum mechanical theory [25] (that is the generalisation of Einstein's theory of relativity), completed by Feynman's contribution [26] and new developments from recent years by Horwitz and Piron [27].

Thus, the parameter  $\tau$  describes the evolution of the system and therefore, it plays the same role as three-dimensional space in Newton's theory, that is not an observable, in contrast to  $t$ , which is the geometrical time, which is an observable. The parameter  $\tau$  was interpreted as proper time of the particle by Stückelberg and Feynman and as historical time by Horwitz and Piron. For a free particle at rest, the proper time and geometrical time can be identically, but, for a compound system in interaction, the proper time for each of the constituent particles cannot also be the proper time of the centre of mass of the system, contrary to the geometrical time. As Stueckelberg demonstrated, the inversion  $\tau \rightarrow -\tau$  imposes automatically the transformation  $\alpha \rightarrow -\alpha$ , because proper time reversed relative to "real" time transform a particle in antiparticle. Consequently, the internal inversion  $\chi \rightarrow -\chi$  transforms a particle in the corresponding mirror particle, and  $\tau \rightarrow -\tau$  represents the transformation of a particle in its antiparticle.

In order to put in accord the predictions with the observed interaction properties, only particles of the same kind have attractive interactions and give bound states (particle – particle, mirror particle – mirror particle, etc.), otherwise they give non-bound states. All interactions between the mirror particles must be the same as between our particles, with but all this would be in fact unobservable, because the only interaction that connects them is gravitational only in rest, will exist different propagators associated to particles and mirror particles, respectively. Thus, the complete image of the Nature could be:



The status of our present theoretical group structure of the nature considering the known interactions is summarised as follows:

SU(3)<sub>c</sub>

QCD (strong interaction),

$SU(2) \times U(1)$

Theory (electro-weak),

General Relativity (gravitation).

It is not included in the current standard

model

and a set of minimum 17 parameters is needed, all of unknown origin. This group must be modified by the presence of corresponding mirror particles: So, two terms must exist in the Lagrangian, one for the known particles, and the other term for their mirror particles:

$$[SU(3)_C \otimes SU(2) \otimes U(1)] \otimes [SU(3)_C^m \otimes SU(2)^m \otimes U(1)^m].$$

The Higgs mechanism, for example, can furnish two vacua, and in respect to the vacuum expected values, the parity can be broken or unbroken. This fact can be also a clue to understand the low value for cosmological constant.

## REFERENCES

- 
- 1 Feza Gursey, in Lectures of the Istanbul Summer School in Theoretical Physics, Ed. F. Gursey, Gordon and Breach, New York, 1964, 365-389
  - 2 Christian Fronsdal, Rev. Mod. Phys. 37 (1965) 221, and some later published papers
  - 3 W. I. Fushchych, I. Yu. Krivshy, Nucl. Phys. B7 (1968) 79
  - 4 E. P. Wigner, Rev. Mod. Phys. 29 (1957) 255, see page 259
  - 5 E. P. Wigner, Ann. Math. 40 (1939) 149
  - 6 S. Weinberg, Rev. Mod. Phys. 61 (1989) 1
  - 7 N. Straumann, astro-ph/9908342, astro-ph/0203330
  - 8 L. B. Okun, hep-ph/0112031, hep-ph/0112032
  - 9 A. D. Dolgov, A lecture presented at the 4-th Colloque Cosmologie, Paris, 1997, astro-ph/9708045
  - 10 S. E. Rugh, H. Zinkernagel, hep-th/0012253
  - 11 S. M. Carroll, <http://www.livingreview.org/lrr-2001-1>, astro-ph/0004075
  - 12 S. A. Bludman, M. A. Ruderman, Phys. Rev. Lett. 5 (1977) 255
  - 13 D. V. Ahluvalia-Khalilova, I. Dymnikova, Int.J.Mod.Phys.D12, 2003, 1787 hep-ph/0305158
  - 14 see I. Dymnikova, gr-qc/0112052 and reference therein
  - 15 F. Mansouri, Coral Gables Conference 2001:, Coral Gables, Fort Lauderdale, Florida, 12-16 Dec 200, AIP Conf. Proc. 624, 42-50, 2002, Phys.Lett. B 538:239-245, 2002 (hep-th/0203150)
  - 16 T. D. Lee, A lecture given at the Int. Symp. In Honour of Boltzmann's 150<sup>th</sup> birthday, febr. 1994, CU – TP – 626
  - 17 T. D. Lee, C. N. Yang, Phys. Rev. 104 (1956) 256
  - 18 A. Salam, Nuovo Cim. 5 (1957) 299
  - 19 I. Kobzarev, L. Okun, I. Pomeranchuk, Sov. J. Nucl. Phys. 3 (1966) 837
  - 20 S. L. Glashow, Phys. Lett. 167 B (1986) 35
  - 21 Matej Pavšič, Int. J. Theor. Phys. 9 (1974) 229
  - 22 R. Foot, H. Lew, R. R. Volkas, Phys. Lett. B272 (1991).67
  - 23 R. Foot, H. Lew, R. R. Volkas, JHEP 0007 (2000) 032
  - 24 R. Foot, hep-ph/ 0207175 and references cited therein
  - 25 E. C. C. Stüeckelberg, Helv. Phys. Acta 14 (1941) 372, 14 (1941) 588, 15 (1942) 23
  - 26 R. P. Feynman, Phys. Rev. 76 (1949) 749
  - 27 L. P. Horwitz, C. Piron, Helv. Phys. Acta 46 (1973) 316