

WAVEFUNCTIONS FOR A RELATIVISTIC ELECTRON IN SUPERSTRONG MAGNETIC FIELDS

GH.DUMITRESCU

Grupul Scolar Industrial Toma N. Socolescu Ploiesti

(Received May 30, 2003)

Abstract. In the past decade few authors attempted to search interesting features of the radiation of a specific neutron star: the magnetar. In this paper we investigate some features of the relativistic electron wavefunction in a strong magnetic field as occurs in a magnetar atmosphere. We have derived the two counterparts of the wavefunction, the spin dependent and the spin independent ones for the Johnson-Lippmann wavefunctions.

Keywords: magnetars, relativistic electrons, high magnetic fields

I. INTRODUCTION

Magnetars are isolated neutron stars with surface dipole field strengths of $\sim 10^{15}$ Gauss, much higher than the $\sim 10^{12}$ Gauss of ordinary pulsars, and they manifest themselves in the form of soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs). Magnetars may be one possible central engine for gamma-ray bursts (GRB), in which case they would put out an magnetohydrodynamic wind or jet which would, decaying as a power law, last much longer than the standard GRB gamma-radiation. X-ray pulsars are strongly magnetized neutron stars in close binary systems. Their X-ray radiation is formed in the magnetic polar caps which are heated by accretion of the matter supplied by the binary companion, or in accretion columns above the polar caps.

For some reasons, which we will present in the following, it is interesting to study the processes which can occur in the magnetic field of the magnetars, especially those in what the electrons are involved. The motion of an electron in the strong magnetic fields was studied in different hypothesizes in order to estimate the cross section of the interactions which may occur between the electron and the photon. One of these interactions is the inverse Compton scattering (ICS), which plays a significant role in the magnetospheric physics of the strongly magnetized stars [1]. Relativistic electrons accelerated above the polar cap can Compton scatter off the thermal radiation from the neutron star surface, introducing high energy gamma rays that can power pair cascade out of the atmosphere of the magnetars. It must be mentioned that neutron stars are born very hot, and cool down to surface temperatures of about 1,000,000 K during the first thousands years of their life, and to about 100,000 K in the next million years. Thermal-like radiation from such objects can be studied with the X-ray observatories.

In their paper Gonthier et al. found deviations between the "exact" cross section for the Compton process of the ultra relativistic electrons in super strong magnetic fields ($B=100$ units in $B_{cr} = m^2 c^3 / e \hbar = 4.414 \times 10^{13}$ Gauss units) and the Klein-Nishina cross section [1]. A classical electron gyrating in a magnetic field satisfies $\dot{p} = ev \times B / c$ where $p = \mathbf{g} m v$. Substituting $\dot{p} = \mathbf{w} \cdot p$ and $v = \mathbf{w} \cdot r$ in the equation above and canceling factors of \mathbf{w} (along with orbital phase) one finds a radius of gyration $r = cp / eB$ where p is the transverse momentum ($p \perp B$). Quantum mechanics implies $r \cdot p \propto \hbar$

in the ground state, thus the semi-classical gyration radius is $r_{gyr} \propto \mathbf{I}_e \left(\frac{B}{B_{cr}} \right)^{-1/2}$, where $\mathbf{I}_e \equiv \hbar / mc$

is the electron Compton wavelength.

The exact cross section is

$$\frac{d\mathbf{s}_{\parallel,\perp}}{d \cos \mathbf{q}'} = \frac{3\mathbf{s}_T}{16\mathbf{p}} \cdot \frac{\mathbf{w}^2 e^{-\mathbf{w}^2 \sin^2 \mathbf{q}' / 2B}}{\mathbf{w}(2 + \mathbf{w} - \mathbf{w}')(\mathbf{w} + \mathbf{w}\mathbf{w}'(1 - \cos \mathbf{q}') - \mathbf{w}^2 \sin^2 \mathbf{q}')} \cdot \frac{1}{l!} \left(\frac{\mathbf{w}^2 \sin^2 \mathbf{q}'}{2B} \right)^l G_{\parallel,\perp} \quad (1)$$

where

$G_{\parallel} = \hat{G}_{noflip}^{\parallel} + \hat{G}_{flip}^{\parallel}$ and $G_{\perp} = \hat{G}_{noflip}^{\perp} + \hat{G}_{flip}^{\perp}$ are functions depending on the incident and scattered photon energies \mathbf{w}, \mathbf{w}' in mc^2 units, on the l number of the Landau level and on the strength of the magnetic field B . σ_T denotes the Thomson cross section. The two indexes “flip” and “noflip” refer to the change or nonchange of the state of the electron spin when the electron changes its Landau level. The differential cross section depends on the final Landau state l' , which will be present in the following. Gonthier et al. called their computed Compton cross section “exact” taking account of the fact that the assumptions used are more complete than those of the other authors: the electron is relativist, i.e. the strength of the magnetic field is about 1 or more and the Landau states between which the electron evolves are larger than 1. The differential cross section was integrated over \mathbf{q}' , the scattered angle of the photon, and Gonthier et al. found that at $B=10$, above the resonance, the scattering process preferably produces photons with parallel polarization, whereas below the resonance, the channel producing perpendicularly polarized photons dominate. This behavior is characteristic of the magnetic-relativistic cross section. Such kind of effects may appear because of the decaying of the excited Landau levels of the electron in a magnetic field.

In order to derive the exact cross section Gonthier et al. have used the Johnson-Lippmann (JL) electron wavefunctions. Although these wavefunctions are not appropriate to describe the spin dependence of the intermediate states of the S-matrix elements they still used it. They argued their choice by the independence of the spin averaged cross section of the intermediate spin states and by the agreement between the exact cross section and the JL cross section at and above the cyclotron resonance. Our attempt is to suggest another appropriate electron wavefunction to describe the behavior in the magnetic fields larger than $B=1$ and with an explicit spin-dependent counterpart.

In the second chapter we derive the counterparts of the electron JL wavefunction. Some ideas about the JL wavefunctions and the Licht field as another solution to the insubstantiality of the JL functions are shown in the third chapter.

II. THE TWO COUNTERPARTS OF THE JL ELECTRON WAVEFUNCTION

The minimal coupling procedure for including an electromagnetic field with 4-potential A^m in Dirac's equation is to replace \hat{p}^m by $\hat{p}^m - q\hat{A}^m$ where q is the charge on the particle. With this assumption the Dirac Hamiltonian is

$$\hat{H} = \mathbf{a}(p + eA) + \mathbf{b}m - e\Phi \quad (2)$$

where \mathbf{a} and \mathbf{b} can be expressed with the Pauli matrices. Choosing the magnetic field along the z-axis

$$A=(0,Bx,0) \quad (3)$$

Dirac's equation depends on only one coordinate, namely x . This choice is called Landau gauge. Therefore the p_y, p_z momenta and the energy are constants of the motion of the electron. One may chooses a wavefunction of the form

$$\Psi(t, x) = f(x)e^{-i\epsilon t + i p_y y + i p_z z} \quad (4)$$

where $\epsilon = \pm$ is the sign of the energy, whose magnitude is e . On inserting the trial solution into Dirac's equation in the form

$$\left(i \frac{\partial}{\partial t} - \hat{H} \right) \Psi(t, x) = 0 \quad (5)$$

one requires

$$\begin{pmatrix} -\epsilon \mathbf{e} + m & 0 & \epsilon p_z & \hat{O}_1 \\ 0 & -\epsilon \mathbf{e} + m & \hat{O}_2 & -\epsilon p_z \\ \epsilon p_z & \hat{O}_1 & -\epsilon \mathbf{e} - m & 0 \\ \hat{O}_2 & -\epsilon p_z & 0 & -\epsilon \mathbf{e} - m \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{pmatrix} = 0 \quad (6)$$

with $\hat{O}_1 = -i \left(\frac{\partial}{\partial x} + \epsilon p_y + eBx \right)$, $\hat{O}_2 = -i \left(\frac{\partial}{\partial x} - \epsilon p_y - eBx \right)$. In (5) ϵ denotes the positive and the negative state of the energy of the electron. Using another variable

$\mathbf{x} = (eB)^{1/2} \left(x + \frac{\epsilon p_y}{eB} \right)$ the equation (6) can be written

$$\begin{aligned} (-\epsilon \mathbf{e} + m)f_1 + \epsilon p_z f_3 - i(eB)^{1/2} \left(\mathbf{x} + d/d\mathbf{x} \right) f_4 &= 0 \\ (-\epsilon \mathbf{e} + m)f_2 - \epsilon p_z f_4 + i(eB)^{1/2} \left(\mathbf{x} - d/d\mathbf{x} \right) f_3 &= 0 \\ (-\epsilon \mathbf{e} - m)f_3 + \epsilon p_z f_1 - i(eB)^{1/2} \left(\mathbf{x} + d/d\mathbf{x} \right) f_2 &= 0 \\ (-\epsilon \mathbf{e} - m)f_4 - \epsilon p_z f_2 + i(eB)^{1/2} \left(\mathbf{x} - d/d\mathbf{x} \right) f_1 &= 0 \end{aligned} \quad (7)$$

and after some calculations one obtains some equations of the same form as Schrodinger's equation for a simple harmonic oscillator:

$$\begin{aligned} \left[\frac{d^2}{d\mathbf{x}^2} + \frac{\mathbf{e}^2 - m^2 - p_z^2}{eB} - (\mathbf{x}^2 + 1) \right] f_{1,3} &= 0 \\ \left[\frac{d^2}{d\mathbf{x}^2} + \frac{\mathbf{e}^2 - m^2 - p_z^2}{eB} - (\mathbf{x}^2 - 1) \right] f_{2,4} &= 0 \end{aligned} \quad (8)$$

The equations above have normalizable solutions if the constant n , defined by,

$$n = \frac{\mathbf{e}^2 - m^2 - p_z^2}{eB} \quad (9)$$

has non-negative integral values. It is convenient to interpret n as the quantum number that determines the perpendicular energy of the particle (referring to the direction of the magnetic field). If one writes

$$2n \mp 1 = 2l + 1 \quad (10)$$

then $(2l + 1)eB$ is the contribution of a simple harmonic motion and seB is the contribution due to the spin. The normalized solutions are the harmonic oscillator wavefunctions

$$v_n(\mathbf{x}) = \frac{1}{(\sqrt{\mathbf{p}} 2^n n!)^{1/2}} H_n(\mathbf{x}) e^{-x^2/2} \quad (11)$$

The $f(x)$ function from (3) can be expressed

$$f(x) = \begin{pmatrix} C_1 v_{n-1}(\mathbf{x}) \\ C_2 v_n(\mathbf{x}) \\ C_3 v_{n-1}(\mathbf{x}) \\ C_4 v_n(\mathbf{x}) \end{pmatrix} \quad (12)$$

where C_1, C_2, C_3, C_4 are constants. Inserting the above function into (6) and taking account of the properties of the wavefunctions for an oscillator one obtains

$$\begin{pmatrix} -\epsilon \mathbf{e}_n + m & 0 & \epsilon p_z & -i(2neB)^{1/2} \\ 0 & -\epsilon \mathbf{e}_n + m & i(2neB)^{1/2} & -\epsilon p_z \\ \epsilon p_z & -i(2neB)^{1/2} & -\epsilon \mathbf{e}_n - m & 0 \\ i[2neB]^{1/2} & -\epsilon p_z & 0 & -\epsilon \mathbf{e}_n - m \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = 0 \quad (13)$$

with $\mathbf{e}_n = (m^2 + p_z^2 + 2neB)^{1/2}$.

Now choose the first two columns of the matrix coefficients and taking account of (3) one obtains the Johnson-Lippmann wavefunction, used by some authors [1], [2], [4]:

$$\Psi_q^k(t, \mathbf{x}) = \frac{e^{-i\mathbf{k}\mathbf{e}_n t + i\mathbf{k}p_y y + i\mathbf{k}p_z z}}{[2\mathbf{k}\mathbf{e}_n(\mathbf{k}\mathbf{e}_n + m)V]^{1/2}} \left[\frac{1+s}{2} \begin{pmatrix} (\mathbf{k}\mathbf{e}_n + m)v_{n-1}(\mathbf{x}) \\ 0 \\ \mathbf{k}p_z v_{n-1}(\mathbf{x}) \\ ip_n v_n(\mathbf{x}) \end{pmatrix} + \frac{1-s}{2} \begin{pmatrix} 0 \\ (\mathbf{k}\mathbf{e}_n + m)v_n(\mathbf{x}) \\ -ip_n v_{n-1}(\mathbf{x}) \\ -ip_z v_n(\mathbf{x}) \end{pmatrix} \right] \quad (14)$$

where $\mathbf{e}_n = (m^2 + p_z^2 + 2neB)^{1/2}$ is the incident electron energy into the static magnetic field B (expressed in $B_{cr} = 4.414 \times 10^{13}$ G units), $\mathbf{k} = \pm 1$ denotes the electron or the positron states, $p_n = (2neB)^{1/2}$ and

$$v_n(\mathbf{x}) = \frac{1}{(\sqrt{\mathbf{p}} 2^n n!)^{1/2}} H_n(\mathbf{x}) e^{-x^2/2} \quad (15)$$

In the expression above $H_n(\mathbf{x})$ are the Hermite polynomials and $\mathbf{x} = (eB)^{1/2} \left(x + \frac{\mathbf{k}p_y}{eB} \right)$.

There are many applications in astrophysics in which the spin counterpart of the wavefunction of the electron in the relativistic regime and in a static magnetic field is used in order to compute the cross sections.

Using some identities $v_n(\mathbf{x})$ can be expressed by a product of two counterparts: a spin counterpart v_s and another one independent of spin $v_l(\mathbf{x})$ (see Appendix).

$$v_s = \frac{2^{\frac{1}{2}(1+s)}}{\left[2^{\frac{1}{2}(1+s)}(1+s)\right]^{\frac{1}{2}}} = \left[\frac{2^{\frac{1}{2}(1+s)}}{1+s}\right]^{\frac{1}{2}} \quad (16)$$

$$v_l(\mathbf{x}) = \frac{(-1)^l e^{x^2/2} \left[\frac{d^l}{d\mathbf{x}^l} (\mathbf{x}+1)e^{-x^2} \right]}{\left(\sqrt{\mathbf{p}} \cdot 2^l \cdot l! \frac{l}{2} \right)^{\frac{1}{2}}} \quad (17)$$

With the expressions above now it is possible to write the two counterparts of the four JL wavefunctions, not in the original way proposed by Johnson and Lippman, but following the procedure proposed by Parle (1987). The Parle-Melrose representation expresses the wavefunction as product of a spin-dependent part and a non-dependent spin part. For $s=1$ and $\mathbf{k}=1$, for example, the JL wavefunction in the Melrose-Parle representation is

$$\begin{aligned} \Psi_q^1(t, x) &= \frac{e^{-i\mathbf{e}t + ip_y y + ip_z z}}{[2\mathbf{e}_n(\mathbf{e}_n + m)V]^{\frac{1}{2}}} \begin{bmatrix} (\mathbf{e}_n + m)v_{l,n-1}(\mathbf{x})v_{s,n-1} \\ 0 \\ p_z v_{l,n-1}(\mathbf{x})v_{s,n-1} \\ ip_n v_{l,n}(\mathbf{x})v_{s,n} \end{bmatrix} = \\ &= \frac{e^{-i\mathbf{e}t + ip_y y + ip_z z}}{[2\mathbf{e}_n(\mathbf{e}_n + m)V]^{1/2}} \begin{bmatrix} v_{l,n-1}(\mathbf{x}) & 0 & 0 & 0 \\ 0 & v_{l,n}(\mathbf{x}) & 0 & 0 \\ 0 & 0 & v_{l,n-1}(\mathbf{x}) & 0 \\ 0 & 0 & 0 & v_{l,n}(\mathbf{x}) \end{bmatrix} \times \begin{bmatrix} (\mathbf{e}_n + m)v_{s,n-1} \\ 0 \\ p_z v_{s,n-1} \\ ip_n v_{s,n} \end{bmatrix} \quad (18) \end{aligned}$$

Melrose argued that Johnson-Lippmann wavefunctions do not correspond to a well-defined spin operator [2]. He also argued that JL wavefunctions have not the same form to the magnetic moment operator but for nonrelativistic electrons. In order to prove his allegation he used the Landau gauge:

$$\mathbf{A} = (0, Bx, 0) \quad (19)$$

magnetostatic field being along z-axis as we already mentioned above.

III. THE LICHT FIELD WAVEFUNCTION FOR THE RELATIVISTIC ELECTRON IN MAGNETIC STATIC FIELD

Deviations from the correct cross sections may appear because of the contributions of decaying the excited Landau states and also of the natural line width of the resonances at the ground state or at the pair production.

Graziani claimed the necessity to describe with good accuracy the QED scattering cross sections in the strong magnetic field, that is to take account to the natural line width of the resonances. The leading-order calculations performed by Daugherty and Harding made no provision for the natural width line of the

resonances [5]. It must be mentioned that Daugherty and Harding computed the Compton cross section of a relativistic electron in the static magnetic field for transitions from ground state to the Landau states with $l > 0$. Their calculus was used by Gonthier et al. It is worthy to underline that the JL states do not describe the behavior of the electron near the resonance ($B=1$). But this is not the case for $B > 1$.

The electron is relativistic in a static magnetic field for $B > 1$ as Duncan have estimated in a recent paper [3]; so in what concerns the choice made by Gonthier et al. [1] that choice was in good agreement with the estimation mentioned.

In order to take into account the contributions of the natural line decay widths Kachelriess proposed to replace the “on-mass shell” condition $p_m p^m = m^2$ to the “off mass-shell” [6]. That is to remove the unstable excited Landau states from the sum of the S matrix to some spectral functions. Using the Licht field did this. For the electron the Licht field is

$$\emptyset(x) = \sum_a \int_0^\infty dE \left(Z_{+,n}^{1/2}(E) b_a(E) \Psi_a^{(+)}(x) e^{-iEt} + Z_{-,n}^{1/2}(E) d_a^+(E) \Psi_a^{(-)}(x) e^{+iEt} \right) \quad (20)$$

where $\Psi_a^{(l)}$ are the energy solutions of the Dirac equation in the presence of the external magnetic field B , $a = \{N, \mathbf{t}, p_y, p_z\}$ denotes the set of quantum numbers needed in order to completely characterize the solutions, and $l = \pm$ distinguishes positive and negative energy solutions. The energy E of the particle is smeared around the on-shell value $E_n = \sqrt{m^2 + 2NeB + p_z^2}$ due to the integration over the spectral functions $Z_{l,n}^{1/2}(E)$ (Here we replaced n from (9) as N). The spectral functions are the generalizations of the wavefunction renormalization constant Z of a stable field and are labeled by the subset of quantum numbers $n = \{N, \mathbf{t}\}$, which enters in the decay-width of the unstable states. In this picture \emptyset describes n -times different particles, i.e. every Landau state with distinct N and \mathbf{t} would be identified as a different particle. Excited states with $N > 0$ are unstable because of the interaction with the photon field. These particles can decay and $Z_{l,n}^{1/2}(E)$ weights the contributions of creating an unstable “ (N, \mathbf{t}) -particle” with energy E and polarization \mathbf{t} . Only the ground state remains stable and for this state one has to set $Z_{l,0,-1}^{1/2} = \mathbf{d}(E - E_0)$. Then, the ground state has the usual LSZ-limit $\Psi_{0,-1} \rightarrow Z_2^{1/2} \Psi_{0,-1}^{out}$ for $t \rightarrow \infty$, where Z_2 is the normal electron wavefunction renormalization constant. The clue of this picture is that describes unstable particles as external and as intermediate states.

In order to use a more appropriate wavefunction for the spin dependence of the intermediate states we suggest the use of Licht field where $\Psi_a^{(l)}$ to be the JL wavefunctions. Therefore one can study the spin dependence behavior of the scattering processes and also to take account of the decay contributions of every intermediate state.

According to Graziani one way to verify if a wavefunction is appropriate to describe the correct spin behavior is to diagonalize the action of $\hat{\mathbf{m}}$ on the wavefunction, [5]. In order to perform this test we shall try into a further paper to investigate the proper choice of the spectral functions $Z_{l,n}^{1/2}(E)$ and of the magnetic moment operator.

CONCLUSIONS

Many authors claimed the necessity to find out the features of the emission of the magnetars in order to prove the existence of such kind of neutron stars. As was presented above Gonthier et al. found that the Compton scattering process in strong magnetic field preferably produces photons with parallel polarization above the resonance and with perpendicular polarization below the resonance ($B=1$). This behavior is characteristic of the magnetic-relativistic cross section. Using JL wavefunctions they also found increasing error with increasing magnetic field between JL cross section and the cross section derived by Sina (1996)

who used Sokolov-Ternov wavefunctions. The increasing error with increasing magnetic field is due to the intermediate states having non-zero momentum. The Licht field expressed using the JL wavefunctions might be an appropriate wavefunction to take account of the intermediate states, which will be prove in a further paper; this will be done by the derivation of the Compton cross section and the corresponding numerical estimation. This calculus will lead to the estimation of the intensity of the emission of the magnetar by the inverse Compton scattering. In the same time the spin dependence of the Licht field proposed above may reveal new features of the decaying of the excited Landau levels.

APPENDIX

In order to separate the spin counterpart of the electron wavefunction we must take account of the next relations:

$$n = l + \frac{1}{2}(1+s) \quad (\text{A1})$$

$$2^n = 2^l 2^{\frac{1}{2}(1+s)} \quad (\text{A2})$$

$$n! = \left[l + \frac{1}{2}(1+s) \right]! = l!(l+1)^{\frac{1}{2}(1+s)} = l! \left[l \cdot \frac{1}{2}(1+s) + 1 \right] \quad (\text{A3})$$

For $l > 10$

$$n! \cong l! \cdot \frac{l}{2}(1+s) \quad (\text{A3}')$$

$$\begin{aligned} H_n(\mathbf{x}) &= (-1)^n e^{x^2} \left(\frac{d^n}{d\mathbf{x}^n} e^{-x^2} \right) = (-1)^l (-1)^{\frac{1}{2}(1+s)} e^{x^2} \left(\frac{d^l}{d\mathbf{x}^l} \cdot \frac{d^{\frac{1}{2}(1+s)}}{d\mathbf{x}^{\frac{1}{2}(1+s)}} e^{-x^2} \right) = \\ &= (-1)^l (-1)^{\frac{1}{2}(1+s)} e^{x^2} \left(\frac{d^l}{d\mathbf{x}^l} (-2\mathbf{x}e^{-x^2})^{\frac{1}{2}(1+s)} \right) = (-1)^l \cdot 2^{\frac{1}{2}(1+s)} e^{x^2} \left(\frac{d^l}{d\mathbf{x}^l} \mathbf{x}^{\frac{1}{2}(1+s)} e^{-x^2} \right) = \\ &= (-1)^l \cdot 2^{\frac{1}{2}(1+s)} e^{x^2} \left(\frac{d^l}{d\mathbf{x}^l} (\mathbf{x}+1) e^{-x^2} \right) \quad (\text{A4}) \end{aligned}$$

where $s = \pm 1$ denotes the spin states. Everyone of the identities expressed above take account of the (A1) and of the values of s . Inserting 2^n , $n!$ and $H_n(\mathbf{x})$ from (A2), (A3)', and (A4) in (11) one obtains

$$v_n(\mathbf{x}) = \frac{(-1)^l 2^{\frac{1}{2}(1+s)} e^{x^2} \left[\frac{d^l}{d\mathbf{x}^l} (\mathbf{x}+1) e^{-x^2} \right]}{\left[\sqrt{\mathbf{p}} \cdot 2^l \cdot 2^{\frac{1}{2}(1+s)} l! \frac{l}{2}(1+s) \right]^{\frac{1}{2}}} e^{-x^2/2} \quad (\text{A5})$$

REFERENCES

1. Gonthier P.L., Harding A.K., Baring M.G., Costello R.M. and Mercer C.L., *Astrophysical Journal*, in press, Vol.541, September 20, 2000, arXiv:astro-ph/0005072.
2. Melrose D., *Quantum Plasmadynamics Book*, RcfTA Director, Prof_Don Melrose, html http://www.physics.usyd.edu.au/theory/ppl/jpegs/melrose_alt.jpg.
3. Duncan R.C., arXiv:astro-ph/0002442 v1 23 Feb 2000.
4. Daugherty J.K. and Harding A.K., *THE ASTROPHYSICAL JOURNAL*, **309**; 362-371, 1986 October 1.
5. Graziani C., *THE ASTROPHYSICAL JOURNAL*, 412, 351-362; 1993 July 20.
6. Kachelriess M., *Phys. Rev. D*53, 974(1996) and hep-ph/9601229.
7. Harding A.K., arXiv: astro-ph/0304120 v1 7 Apr 2003.