THE DRESSED STATES OF AN ELECTRON IN THE MODIFIED PŐSCHL-TELLER POTENTIAL

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Abstract. We present results in high-frequency limit of Floquet theory for the modified Pöschl-Teller potential. The energies of four bound states, and the widths of the first two, are calculated as functions of the electric field parameter α_0 (the classical free electron quiver motion amplitude). The mechanism of evolution of a resonance to a pair of bound and antibound states of the dressed potential is illustrated for two resonances.
The similarity of the results with the equivalent ones corresponding to two zero-range potential (δ and double δ-function potential) is discussed.

Key words: Pöschl-Teller potential, HFFT, dressed potential, resonances, light induced-states

1. INTRODUCTION

The unusual behavior of an atomic system interacting with a monochromatic electromagnetic field, as predicted by the Floquet theory, has attracted a large number of studies, both theoretical and experimental. Also, the Floquet solutions are used, in the multistate Floquet theory, for the description of the time evolution of an atomic system interacting with a laser pulse.

At high frequency and intense fields, the Floquet theory takes the form of the high frequency Floquet theory (HFFT), developed by Gavrila and Kaminski [1], [2]. During the last 15 years, a large number of systems were studied in the frame of Floquet theory, at finite frequency or in the high frequency limit. Although some of the existing work refers to realistic, three dimensional systems, the vast majority of them concerns the simpler one dimensional models; their study continues to be of interest in connection with time dependent problems.

In section 2 we present a few elements of Floquet theory. In section 3 several results for one-dimensional systems are briefly reviewed. The remainder of the paper is devoted to the modified Pöschl-Teller potential. In section 4 we reproduce the analytical solution of the stationary Schrödinger equation associated to the modified Pöschl-Teller potential and in section 5 we calculate the energies of several bound states, anti-bound states and resonances of the dressed potential, in the range (0-20) for α_0; we present several light induced states and their connection with the resonances of the dressed potential. Also, for several finite frequencies the ionization rates of the bound states are calculated in the first order of $1/\omega$. Our conclusions are presented in the section 6. Atomic units are used in all the equations.

2. ELEMENTS OF FLOQUET THEORY

We consider an electron interacting with an atomic potential in a monochromatic electromagnetic field, of intensity I, treated in the dipole approximation. The corresponding time-dependent Schrödinger equation, in the Kramers-Hennberger reference frame is [2]:

$$\mathrm{i}\frac{\partial \Psi(\mathbf{r},t)}{\partial t} = \left[\frac{\mathbf{P}^2}{2} + V(\mathbf{r}+\boldsymbol{\alpha}(t))\right]\Psi(\mathbf{r},t) \tag{1}$$

In the previous equation $\boldsymbol{\alpha}(t)$ is the trajectory of the classical electron in the electromagnetic field; for the case of linear polarization the motion is an oscillation with the frequency ω of the field and constant amplitude, $\alpha_0 = \sqrt{\mathrm{I}}/\omega^2$. According to the Floquet theory, the equation has particular solutions with the structure:

$$\Psi(\mathbf{r},t) = \mathrm{e}^{-\mathrm{i}Et}\Phi(\mathbf{r},t) \tag{2}$$

where $\Phi(\mathbf{r},t)$ is a periodic function of time, with the period $T = \dfrac{2\pi}{\omega}$ of the field. Equation (1) becomes an infinite system of coupled differential equations for the Fourier components $\Phi_n(\mathbf{r})$ of $\Phi(\mathbf{r},t)$.

$$-\frac{\hbar^2}{2}\Delta\Phi_n(\mathbf{r}) + \sum_{m=-\infty}^{\infty} V_{n-m}(\mathbf{r})\Phi_m(\mathbf{r}) = (E+n\omega)\Phi_n(\mathbf{r}) \tag{3}$$

where $V_n(\mathbf{r})$ are the Fourier components of $V(\mathbf{r}+\boldsymbol{\alpha}(t))$.

$$V_n(\mathbf{r}) = \frac{1}{T}\int_0^T dt' V(\mathbf{r}+\boldsymbol{\alpha}(t'))\mathrm{e}^{in\omega t'} \tag{4}$$

If the Floquet system of equations is solved with the ionization boundary conditions

$$\Phi_n(\mathbf{r}) \xrightarrow[r\to\infty]{} \frac{e^{ik_n r}}{r}, \quad k_n = \pm\sqrt{2(E+n\omega)} \tag{5}$$

the solution E, called quasinergy, takes complex values, whose imaginary part is interpreted as the ionization rate in the corresponding Floquet state. If the sign of the momenta k_n is chosen as

$$\mathrm{Im}(k_n) > 0, \quad \text{if} \quad \mathrm{Re}(E) < 0, \quad \mathrm{Im}(k_n) < 0, \quad \text{if} \quad \mathrm{Re}(E) > 0 \,, \tag{6}$$

the Floquet solution is called ‘physical’; otherwise, the Floquet solution is ‘nonphysical’, or ‘shadow’.

In the limit $\omega \to \infty$ the Floquet system of equation reduces to the stationary Schrödinger equation

$$-\frac{\hbar^2}{2}\Delta\Phi(\mathbf{r}) + V_0(\mathbf{r})\Phi(\mathbf{r}) = E\Phi(\mathbf{r}) \tag{7}$$

corresponding to the so-called 'dressed potential', the zeroth order Fourier component of $V(\mathbf{r} + \boldsymbol{\alpha}(t))$, which is its average over one period.

Gavrila and Kaminski [1], [2] have developed the high frequency Floquet theory, which allows expressing the solutions of the Floquet system of equations as a series of powers of $1/\omega$. In the limit $\omega \to \infty$, at fixed value of the parameter α_0, the HFFT quasienergies coincide with the eigenvalues of the equation (7); if higher order terms are considered, the quasienergies become complex. The imaginary part of a quasienergy, in the first order is expressed as a series

$$\Gamma_n = \sum_{E+M\omega>0} \Gamma_n^M = 2\pi \sum_{E+M\omega>0} \left| \int d\mathbf{r}\Phi_n(\mathbf{r}) V_M(\mathbf{r}) \Phi(\mathbf{r}, E + M\omega) \right|^2 \tag{8}$$

In the previous equation, $\Phi_n(\mathbf{r})$ is the n-th bound state of the dressed potential, and $\Phi(\mathbf{r}, E + M\omega)$ is the continuum wavefunction of the dressed potential, corresponding to the energy $E + M\omega$, normalized in the energy scale.

As the new laser sources are capable of providing pulses of intensities of the order of atomic unit, far above the perturbative domain, the Floquet theory is a very powerful tool and is largely used in the study of the interaction of atoms with very intense electromagnetic fields. One of the most important prediction of the Floquet theory is the so called 'atomic stabilization', which is the property of the ionization rates to decrease with the increasing of the electromagnetic field intensity; the experimental evidence of the atomic stabilization was found in two experiments, performed at FOM Amsterdam [3], [4] [5].

In the case of an realistic laser pulse, the solution of the time-dependent Schrödinger equation can be expressed, in the frame of the Multistate Floquet Theory as superpositions of Floquet states; a review of the Floquet theory and of its recent developments is presented in Gavrila's paper [6].

3. PREVIOUS WORK

There is a large number of Floquet calculations in the literature for the one-dimensional systems; the study of the one-dimensional systems has the advantage that the numerical difficulties associated with solving the Floquet problem are fewer than in the three dimensional cases. At the same time, the qualitative behavior of theses systems might be used in order to describe the realistic three-dimensional case.

One of the systems studied was the Gaussian potential viewed as a rough model for the potential of atomic ions like Cl^-. The Floquet problem for this potential was solved by Bardsley and Comella [7], Dörr and Potvliege [8], Yao and Chu [9]; Marinescu and Gavrila [10] also have calculated the quasienergy of the ground state and the first light induced state, and have studied the limits of validity of the HFFT. Their results show that, at sufficiently large α_0, the HFFT results are in extremely good agreement with the exact ones, even for the relatively small frequencies.

The scattering Floquet problem attached to the one-dimensional rectangular well was solved by Kaminski [11]. Using a slightly modified version of Kaminski's method, Fearnside *et al.* [12] and Day *et al.* [13] have solved the ionization Floquet problem for the same system.

The one-dimensional δ-function potential attracted a large number of studies. The HFFT results were obtained by Grozdanov [14], Sanpera [15], and, recently, by us [16]. We have used a very efficient method developed by Chirila [17]. Also, Dörr *et al.* [18] solved the Floquet ionization problem at finite frequency, using a version of Kaminski's method. In [19] we have presented the exact Floquet results at several frequencies and the HFFT results for a Gaussian potential and for the one-dimensional δ-function potential. Our conclusion was that the convergence of the Floquet results to the HFFT at increasing frequencies strongly depends on the potential, being extremely poor in the case of the δ-function potential. Another example of zero-range potential is the double δ-function potential, recently studied in the high frequency approximation [20].

In order to describe hydrogen-like atomic systems long range one-dimensional potentials were used. The so-called soft-coulomb potential,

$$V(x) = -\frac{1}{\sqrt{x^2 + a^2}}\,, \tag{9}$$

was widely used in connection with the time-dependent problem (see, for example [21]). The energies of the first two bound states of the dressed potential were calculated by Bhatt *et al.* [22]. Wells, Simbotin and Gavrila [23], [24] have calculated the Floquet map for a slightly different soft-coulomb potential,

$$V(x) = -\frac{a}{\sqrt{x^2 + a^2 e^{-\frac{x^2}{a^2}}}} \tag{10}$$

The trajectories of the Floquet states that emerge from the bound states of the free system have been obtained, and, also, the quasienergies of several light induced states were calculated. Using the numerical solution of the time-dependent Schrödinger equation, the physical reality of the light induced states was proved. A comprehensive Floquet map for the potential (9), at several finite frequencies and in the high-frequency approximation was calculated by Constantin [25], using a code developed by Simbotin and Marinescu.

Recently, another short range potential, the modified Pöschl-Teller potential, which is also the subject of the present work, was studied in the frame of Floquet theory. Yasuike and Someda [26] have determined the quasienergy of the ground state and of the first light induced state at several frequencies above and below the zero-field ionization threshold; Wassaf *et al.* [27] have calculated the quasienergy of several light induced states lying very close to the continuum, at low frequency, and showed their influence on the photoelectron spectra obtained at the end of a laser pulse.

4. THE MODIFIED PŐSCHL-TELLER POTENTIAL

The modified Pöschl-Teller potential [28]

$$V(x) = -\frac{V_0}{\cosh^2\left(\frac{x}{a}\right)} \tag{11}$$

was studied long time ago. It is a textbook problem in quantum mechanics (see for example [28], [29]).

The stationary Schrödinger equation attached to this potential has analytical solutions; we reproduce them here briefly.

With the notation $s = \frac{1}{2}\left(\sqrt{8a^2V_0 + 1} - 1\right)$, two linear independent solutions of the Schrödinger equation are:

$$v_1(x) = \cosh\left(\frac{x}{a}\right)^{ika} {}_2F_1\left(-ika - s, -ika + s + 1, 1 - ika, \frac{1 - \tanh\left(\frac{x}{a}\right)}{2}\right), \tag{12}$$

$$v_2(x) = v_1(-x), \quad 1 - ika \neq -N, \quad N = 0,1,2,...$$

If k takes imaginary values $k = \mathrm{i}\kappa_n$, where κ_n is in the discrete set,

$$\kappa_n = (s - n), \quad n = 0,1,2,\ldots,[s] - 1, \tag{13}$$

$v_1(x)$ and $v_2(x)$ obey the relations:

$$v_1(x) = (-1)^n v_2(x), \quad v_{1,2}(x) \xrightarrow[x\to\infty]{} \pm \mathrm{e}^{-\kappa_n x} \tag{14}$$

They describe bound states of the system, of energy $E_n = -\frac{\kappa_n^2}{2}$, and parity $\Pi_n = (-1)^n$. There are no resonances of the modified Pöschl-Teller potential

5. THE HFFT RESULTS

Our numerical calculation, presented in the following, refers to the choice $V_0 = 1, \quad a = 1$ in (11); this choice corresponds to $s = 1$. At zero field intensity the potential has only one even bound state, of energy $E_b = -0.5$. We have numerically determined the solution of the Schrödinger equation (7), with adequate boundary conditions and have found several bound states, anti-bound states and resonances.

We shall describe first the bound states of the system. The energy of the only one bound state at $\alpha_0 = 0$ increases monotonically with the value of the parameter α_0 (see Fig. 1) . With the increasing of α_0, also light induced states appear. The first light induced state, which is odd, emerges as soon as α_0 is different from zero; as α_0 increases, its energy decreases, and then starts to increase again. For $\alpha_0 > 12$ the ground state and the first induced state are quasi-degenerated. We have found other two light induced states of alternating parity, which appear at $\alpha_0 = 4.3$, $\alpha_0 = 10.82$. In Fig. 1 the bound states are denoted as $b_0, \dots b_3$. Also, the energies of four antibound states of the dressed potential are represented; these antibound states will be discussed below.

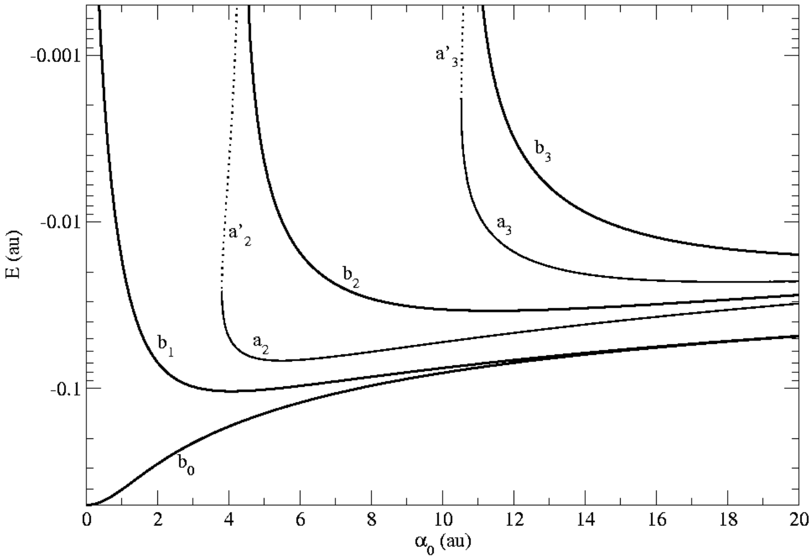

Fig. 1 - The energies of the bound states $b_0,\ldots b_3$ and antibound states a_2, a_3, a'_2, a'_3 of the dressed Pöschl-Teller potential as functions of the parameter α_0

We have calculated the complex energy of two resonances for $\alpha_0 > 3$. In Fig. 2 the trajectories of these resonances, denoted by r_2 and r_3, are represented in the k-plane; their position at $\alpha_0 = 3$ is marked by a star. As we shall see, there is a strong connection between theses resonances and the light induced states b_2 and b_3. As α_0 increases, the resonances move in the complex plane, and, at a certain value of α_0 the pole which correspond to a resonance and its conjugate reach the imaginary axe becoming two antibound states; the corresponding values of the parameter α_0 are marked on the figure. Further they move on the imaginary axe in opposite senses, the pole which moves upward eventually reaching the upper semiplane, thus becoming a light induced state. We could not follow the trajectory of the resonances for $\alpha_0 < 3$ due to fact that the imaginary part of k takes very large negative values.

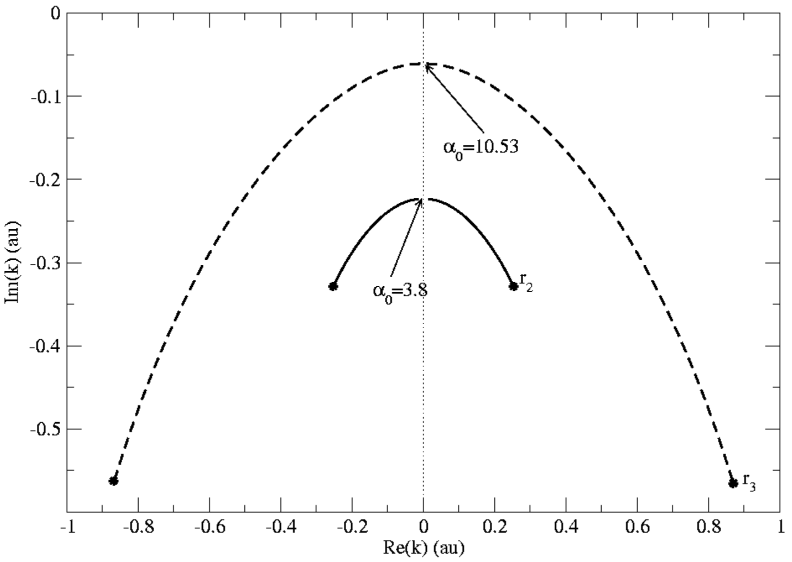

Fig. 2 - The trajectory of the resonances r_2, r_3 and of their conjugate poles in the k -plane

The energies of the antibound states are represented in Fig. 1; they are denoted by a_2, a'_2, and, respectively, by a_3 and a'_3. Our conclusion is that, except for the first light-induced state, which appears at $\alpha_0 = 0$, all the others light induced states emerge, trough an intermediate antibound level from a resonance. The same relation between the resonances, anti-bound and bound states of the dressed potential was observed also for the case of one dimensional δ [16] and double δ -function potential [20], and probably is a common feature of short range potentials.

In Fig. 3 the ionization rates in the first order of HFFT are represented, for the ground state and the first induced state of the dressed modified Pöschl-Teller potential, for two frequencies: $\omega = 1$ (left graphic) and $\omega = 2$ (right graphic). We mention that the convergence of the series (8) is very fast, less that 10 terms being enough; the same fast convergence was observed [19] for another short-range potential, the Gaussian potential; this is in the contrast with the case of zero range potentials, as δ [19] and double δ -function potential [20], where the convergence is extremely slow, hundred of terms being required.

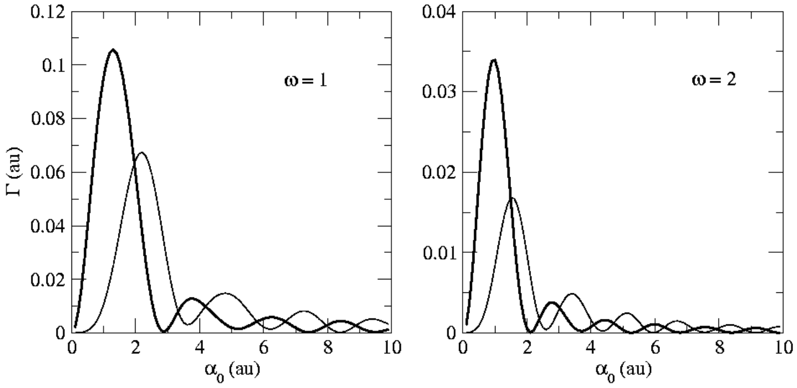

Fig. 3 - The ionization rates in the first order of HFFT of the ground state (thick line) and first excited state (thin line) of the dressed Poschl-Teller potential

One notices the so called ‘atomic stabilization’ phenomenon, which is the very fast decrease, although in an oscillatory manner, of the ionization rates with α_0, for α_0 beyond a certain value. Another remark is that at a given value of α_0, the ionization rate for $\omega = 2$ are always smaller than the corresponding ones for $\omega = 1$, which is also in agreement with the prediction of the Floquet theory.

6. CONCLUSIONS

We have calculated the energy of the ground state and of the first three light induced states of the dressed Pöschl-Teller potential as a function of the parameter α_0. For the ground and first induced state we have calculated the ionization rates in the first order of $1/\omega$, for several frequencies.

By comparing the results obtained with the equivalent ones for the δ and double δ-function potential, we have found similarities between the mechanism of apparition of the light induced states in the two cases; this mechanism seems to be a common feature of the short range potentials.

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